A Simulation Study of Individuals' Price Expectations and Market Processes

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ABSTRACT

This study examines the behaviour of a set of heterogeneous agents whose approach to decision-making is based on individual Information Processing Rules. These reflect their attitudes to the information in their recent observations relative to their past experience. The characteristics and influence of these agents are explored in a simulated speculative market which is order-driven. Using a variant of Bayesian sequential updating, agents base their predictions on a general model that includes the random walk and incorporates some features of Technical Analysis.

After an overview of the short and long-term behavioural implications of the rules as they affect individual agents, the compositional behaviour of different types is examined. These types interact within an evolutionary structure in which agents are replaced in the market according to increasingly rigorous criteria of financial success. The continuous process of renewal that eventuates indicates that increased accountability requirements, while resulting in greatly increased profits for some individuals, tend to generate an unstable market, greater risk and increased volatility.

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Chapter 1

Information in the Stock Market

1.1 Introduction

In October, 1997, the news presentations of the media were being dominated by reports of rapid falls in Asian stock market prices. Images on television screens showed traders in many centres watching electronic displays with anxious expressions. What was chiefly interesting to all these people was information. Traders both large and small needed information on which to base decisions as to what action would be the best to take in the unusual situation facing them.

Grossman [29] classifies agents in a market as being 'informed' or 'uninformed', the distinction being based on the cost of information. A dictionary defines information as "that of which one is apprised or told . . . news" ([53]). In the stock market context, an example of this news could be the price of the most recent transaction or yesterday's volume of trade. A price or volume, however, is only significant within a context. Such figures need to be interpreted and many market advisers do, in fact, attempt to do this.

How then should the information that is to be the input to a decision-making process be defined? The economic agent making this decision is by no means a passive entity. His perception of a market event is a subjective action in which his personal attitudes and interests are important. A now obsolete meaning of the word 'information' implied the giving of a form or shape to an event, which is precisely what an individual agent does when he observes a figure or set of figures or when he exchanges views with other traders. Before any action is taken, the event is transformed and given meaning as it is related to the individual's own unique experience and his own judgment.

What the agent is doing has been described by Zellner [69] as an *Information Processing Rule.* Such a rule links the raw facts, that is, the new data which, together with a previously held opinion, are designated by Zellner as the input information, with the formation of an updated opinion. This opinion then constitutes the posterior distribution or output information on which the agent's next decision is based. If these rules vary for different individuals then the output information that will be produced will vary considerably. Zellner derived an optimal rule which, in fact, describes Bayes' Theorem, the criterion of optimality being that the input information should be equal to the output information. This criterion he calls *the Information Conservation Principle.* It is this output information which is used in decision making.

However, if the input from new observations and from experience of the past are rated differently by agents and their relative importance reflected in different weights, then the Information Processing Rules will differ from Bayes' Theorem. Where this is the case, a number of traders participating in a market will have an equal number of personalised rules for processing information. The majority of these rules will not be optimal in Zellner's sense. They have even been described by some as 'irrational'. The many subjective distributions of these agents will now govern the decisions made and so create the series of prices observed by the traders. It then becomes the aim of each of these traders to predict the next step in this series so that he may profit by it, or, in the case of the Asian traders mentioned above, take steps to prevent disastrous losses.

This subjective response of each individual trader to information does not take place in isolation, but against the background not only of a market structure which, to some extent, sets the pattern of trading, but also of a population consisting of a large number of other traders whose responses he must attempt to assess within an atmosphere of considerable uncertainty. These form the environment for the trader. How he interacts with this environment will determine how long he survives there and the extent to which he benefits from it. The next Section gives a brief overview of this environment.

1.2 The Stock Market

1.2.1 The Stock

When a company is in need of capital, it can obtain this by borrowing or it may issue equities. The latter, which are the concern of this work, entitle the owner to dividends and give voting rights and other privileges. These are issued in what is called a *primary market.* Liquidity for the investors is provided by the *secondary market*, comprising the Stock Exchange and other trading facilities where stock can change hands at a price which reflects its current value as perceived by the general population of traders.

There are a number of different categories of stock. Some investors may require a safe investment with prospects of a regular dividend income. Prices of these 'blue-chip' stock are characteristically high to buy into, move within a fairly limited range, and are not as a rule volatile. In other words, they are a reliable long-term investment. There are other types of stock that are actively traded and generally considered as a speculation rather than as a solid dividend producing investment. Movement is greater than for blue-chip stock so that profits can be made over a shorter time range. For this type of trading, securities where swings are sufficiently frequent for profits to be made in spite of transaction costs are the most attractive. Moreover, it is easier to predict effectively over a shorter time horizon. As Keynes [37] points out, the formation of long-term expectations is not an easy matter in modern markets.

1.2.2 The Market Architecture

Trading systems can be divided into two broad categories. Some depend on the matching of buy and sell orders, either by brokers or by computer.

These are known as *order-driven markets.* Others, the *price-driven* markets, are organised by competing specialists or market makers, who quote bid and ask prices for the stocks for which they are responsible. Specialists provide traders with immediacy, their task being to ensure a fair and orderly market. They have a greater insight into market conditions, a situation that assists them in the task of rendering the markets less volatile. In the process, of course, they may influence prices to some degree. Many stock exchanges are a mixture of these two types, being primarily matching markets but with specialists helping to provide a smooth and continuous market.

Since changes to the laws governing the trading of securities in various countries in the '70s and '80s, there has been a steady updating of market systems, with interaction between different national markets encouraged by the extension of computer technology and an opening up to overseas investments. In this context, the very structure of the exchanges themselves began to change. For example, the matching of buy and sell orders by computer facilitated the handling of the increased flow of orders generated by the various reforms and regulations that were taking place. The NYSE is notable for its retention of an open outcry system on a market floor with trading posts. Even here, however, there is a wide supplementary use of screen-based technology. The new computer technology has also served to connect national and international markets with each other and with the brokerage houses.

The markets aim to be highly informative. Automated quotation systems waste no time in displaying details for the most actively traded stocks. These details include the closing price for the previous day, the current best bid and ask prices, the latest transaction price and the cumulative volume of trading for the current day. The increase in computerisation and the resulting widespread availability of information has meant that large financial institutions no longer have the research advantage over other investors.

1.2.3 The Order Flow

Prospective traders, having processed the information at their disposal, form their expectations of the price for the next trading period as a basis for the prices at which they will either buy or sell the desired quantities of stock. They then determine their preferences for bid or offer and present the appropriate orders. Agents consider that they have made an informed estimate of what the stock in which they are interested is really worth in the market. If the current trading price conforms to this estimate, the agent will place an order for an immediate sale. This is called a *market order.* Intraday changes in price may now benefit him, but he runs the risk of getting a less favourable price than he had possibly hoped for. On the other hand, if he sees the stock as undervalued or overvalued, he may place a *limit order* to trade at a predetermined price. He is prepared to wait in the expectation that the price will shortly return to the vicinity of his expected value. Limit orders are recorded by specialists or clerks to be held until activated by price movements at some later date. Such orders are placed for varying time periods. Limit orders are frequently supported by *stop loss orders* to reduce the risk associated with an unanticipated change in the direction of the market.

These orders cannot be expected to constitute a continuous flow, yet orders have to be matched and prices negotiated. Nor is this flow of orders independent of the particular trading system that is in operation in the market approached, since this system determines how their orders are transformed into trades. For example, for an agent prepared to "meet the market" a greater degree of certainty is provided when there is a specialist system in place. A theoretical model may not necessarily allow for this unevenness of flow, as the actual prices at which trades are made frequently have more to do with opportunity than with the intersection of supply and demand.

1.2.4 The Agents

After World War II, there was a resurgence of interest in the stock market on the part of private individuals and later of institutions. Participants ranged from long-term investors to speculators working over short time horizons.

One of the theoretical advances of the '80s was the development of the asset market approach to the study of equities. Financial assets came to be regarded as durable commodities. Not only do they yield returns at regular intervals in the form of dividends and other benefits, but can be offered for resale at a future date in the hope of making a capital gain. For the speculative type of market, such an increase in the capital value is the primary consideration for traders. In practice, the increased volatility of such a stock is welcomed by the speculator, who always hopes, in the conditions of uncertainty that prevail, to outpredict his competitors. This type of trader frequently buys stock with the sole aim of reselling it when a favourable opportunity arises. In this type of market, then, the expectations of the traders, formed on the basis of the information available to them, are of paramount importance.

Of growing importance in stock markets are the institutional investors, a group that have increased over the last few decades at the expense of individual traders. Managed funds appear to be of interest to investors largely because, due to the size of their funds, they are in a position to offer their clients a degree of portfolio diversification that would not otherwise be possible, thus diminishing the risks of trading. However, fund managers as a whole have consistently failed to outperform the indices in many studies. There has been some concern about the power of institutional investors and the possible destabilising effects they may have on the market, leading to several commissions and reports in the '70s, and in some countries to regulations that have limited the proportion of equities in institutional portfolios. Others have found no evidence that their trading was causing price instability. On the whole, institutional investors appear to compare with other investors both in the degree of their aggressiveness and their rationality.

1.3 Survey of the Literature

1.3.1 Information Processing and Efficiency

As early as 1900, Bachelier [4] had put forward several stochastic models for speculative prices which implied the efficient use of information. He described the market as a fair game in which expected profits would be zero using in particular a Brownian motion model. Three decades later, the events of the depression of 1929 stimulated a renewed interest in financial markets. Based on variations of autoregressive models, learning models were developed. These stressed the formation of expectations and the adjustment of opinions as economic agents revised these expectations to learn from their errors.

These developments culminated in a theory known as the Efficient Market Hypothesis, (EMH), associated with the asset market approach. First stated as a formal theory by Samuelson [56], this theory makes two basic assumptions. The first of these is that information is processed by all agents in a rational manner. Muth [52], who first proposed the theory of Rational Expectations, asserted that expectations are continuously updated by agents according to their interpretation of the flow of new information with the aim of optimising their utility functions. Aggregation of individual information sets results in a distribution that is the same as that defined by the economic theory, so that market expectations reflect the mathematical expectations of the model. Consequently, any errors made will bring about movements in price that are contrary to expectations and arbitrage will occur that will restore equilibrium, so that errors will not become systematic. This argument, put forward by Fama [17] and others, implied that any agents not acting rationally could make no lasting impression and, moreover, would soon be driven from the market by their financial losses.

The second assumption of the EMH is that all generally available information is fully reflected in the most recent price. This is believed to occur within a discrete interval of time, that occurring between price quotes, so that adjustment can be considered to be immediate. Thus there can be no patterns in past prices that would enable a trading system to be developed that could result in extraordinary profits.

The information referred to in this theory is therefore restricted to what is frequently called *the underlying fundamentals*. These include information regarding dividends, the prospects and management of the industry or company concerned or perhaps government policy as it affects it. Much of this information is generally known by all agents and incorporated in their expectations which are then reflected in the current price. Other information is unexpected and occurs randomly. The basis for the Random Walk

Theory of prices is that it is this unexpected information that is responsible for changes in price. Next period's price differs from the current price only by a random disturbance, implying that stock prices are unpredictable. Considerable research, including that of Meese and Rogoff [51] has produced results showing that the Random Walk model outpredicts many structural and other models. I had found similar results in my analysis, Johnson [34], of the foreign exchange market for the New Zealand dollar after its float in 1985. Kokaua's [40] study involving structural economic measurements concurs as well.

1.3.2 Accounting for Volatility

It is widely believed by many market professionals and also by many theorists that stock prices do not always reflect the fundamentals, especially in the more speculative type of stock. Shiller [58] [59] showed that price series were generally more volatile than would be expected on the basis of a rational assessment of economic factors affecting the stock, that is, if expectations were based on the discounted present value of actual dividends as forecast from the generally available economic information. It seemed that some other influence was affecting market decisions and generating this volatility. Some attempts were made to develop models allowing for bubble effects within the framework of the efficient market model, but these were not particularly convincing. Some of the work in this field is surveyed by Kirman [38].

Where there is increased volatility, the prediction of price changes becomes correspondingly more difficult. Good forecasting is of great value for a speculative trader but studies in economics have shown a generally poor performance, especially in the anticipation of turning points. (See references quoted by Kling [39]).

A variety of different approaches have since been used to address this question of excess volatility. It is frequently implied that not all investors are acting in a fully rational manner since their beliefs are formed on the basis of factors other than news of fundamentals. Shleifer and Summers [61] and also de Long et al. [15] analyse two groups which they call rational speculators or arbitrageurs on the one hand, and noise traders who trade on non-fundamentals on the other, and whose judgments are subject to systematic biases. Although the arbitrageurs do trade against these, there is nevertheless a degree of risk attached to such trading that prevents it from completely countering the effects of what the authors designate as uninformed trading and so bringing the market back to a fundamentalist equilibrium. This same risk may, in some cases, make their trading profitable. In any case, the interaction between the two groups fosters volatility in the market.

Samuelson had shown that an assumption of homogeneous expectations resulted in market efficiency. Since this time, a considerable amount of literature has dealt with the effects of the relaxation of this assumption. Many of these authors have divided market operators into two or more groups according to different methods of classification. Some of these used differing theoretical approaches to decision making, others specified limitations on the extent of the information set. Shiller [59] himself discussed the effects of social movements and influences, and proposed a model categorising agents under the headings of "smart money" and "ordinary investors". The first are represented as responding efficiently to the generally available information regarding the returns on investment, the second as overreacting or following popular trends or fads.

Black [7] declares that, because people trade on noise, the noise factor cumulates to such an extent that traders must resort to the use of rules of thumb which they do not realise are too simple. Heiner [32] attributed a similar result to a Competence-Difficulty Gap. Whatever the assumed origin of such rule-driven behaviour, there is evidence of an increase in this type of trading over the last twenty years. (See Coninx [14]). Since the '80s there has been much less attention given to fundamentals and much more to the methods of *Technical Analysis.* Work by Frankel and Froot [23], in an attempt to bring theory closer to the views of market participants, based a model of foreign exchange, also an asset market, on the activities of the two groups, Fundamentalists and Chartists. Weighting of the predictions of these two groups was updated in a rational Bayesian manner to form the expectations of traders in the market. Goodhart [27] also modelled these two groups and described the conflict between them. The model of Frankel and Froot was further developed by Kirman [38] to illustrate the role of non-fundamentalist traders in generating bubble-like phenomena.

In some studies, heterogeneity is seen in the Information Sets themselves rather than in the agents' approach. The uninformed agents of Grossman and Stiglitz [29] must depend on the price system alone, whereas the remaining agents have spent valuable resources on obtaining more comprehensive information. In their model, individuals are considered equal in their ability to process information and, moreover, are considered to do this in a rational manner. Figlewski [19] [20] also approached the question of market efficiency by introducing diversity of information into his models, both in groups and individually. In these models, expectations and forecasting ability were affected by the different quality of the information available to each trader, information that was then weighted by the size of the agent's investment. Goodhart [27] suggested that differences between short and long-term patterns in exchange rates arise from the use of different information sets.

A further step was to depict agents as individuals differing in their beliefs and approach to processing the information available, whether this information was identical for all or not. This led to models attempting to formalise the way in which agents may vary their opinions. This was done by Lad [41] who describes agents whose decision-making is subjective and each of whom places a different emphasis on more recent as opposed to more distant information. Arthur et al. [2] also have a heterogeneous set of agents making their decisions by continually forming and reforming hypotheses which they test in a simulated stock market, enabling them to adapt to an endogenously changing market.

What is known as market sentiment has long been seen as an important influence in speculative markets. Keynes [37] recognised that prices reflect what market opinion, frequently based on the extrapolation of past prices rather than on fundamentals, expects it to be. It is generally believed among market practitioners that it does not pay to ignore the majority sentiment. Swings in prices can be generated by the feedback effects of the exchange of views between traders. Kirman's agents, as do those of de Long et al., take market opinion into account and change their opinions between the two alternatives as a result of their interaction with other agents and according to the degree of their financial gains or losses. Garbade et al. [24] studied the price quotations of other agents as a source of new information leading to the revision of opinions. These authors also included a parameter to reflect the confidence placed by each agent in his estimate of the price relative to those of other agents. In the Information Contagion Model of Arthur and Lane [1], agents supplement the publicly available information by making their decisions sequentially on the basis of a sampling of the decisions of recent participants in a market.

These models are dealing with the interaction between agents that takes place before decision-making, and with its effect on the interpretation of the information received, showing the importance of the feedback effect and the subsequent revision of that interpretation. Renshaw [55] represented population groups in time series by an equation with terms for individual growth with and without interaction, considering both nearest neighbour and more distant interactions. That the degree of interaction has a strong influence on volatility had been stressed by Averintzev [3] in the context of the effect of the microeconomic on the macroeconomic. He also formulated separate functions for the state that was inner directed and that where interactions took place. Some writers have drawn an analogy with the Ising Model of statistical dynamics. A class of economic models were developed by Föllmer [22], which examined the interacting preferences of agents, and were applied in the context of sociology by Haken and others. Föllmer believed that the interacting opinions of economic agents were an important source of volatility. Haken [30] used the Ising model of the ferromagnet as an analogy for the transition probabilities in a model for the formation of opinion in which he included a term for social climate and distinguished between interacting and non-interacting agents. These writers pointed out that just as phase transitions occur when interactions are strong and complex enough, so volatility in a human system was found to occur when a large number of individuals acted in the same direction. This is illustrated in the context of the stock market by the extrapolation methods frequently used by technical traders. If each agent makes his decision independently of others, the law of large numbers will apply and extremes will cancel out, but the correlation between their actions that frequently results from interaction is seen to translate into the rises and falls so characteristic of financial markets.

1.3.3 The Bayesian Approach

The Bayesian method of opinion formation has long been associated with learning models. This method is based on the utilisation of sample information to update current knowledge, which, given the uncertainty of the agent, is expressed as a prior probability distribution. Learning models will then combine this prior with a likelihood function of the observations from which a posterior distribution for the parameters can then be deduced by Bayes' Theorem.

Empirical work using this approach ranges over both structural and time series models and utilises the methodology of sequential updating. In the context of the theory of the communication of information, the use of Bayes' Theorem was suggested by Woodward and Davies [67] and by Cherry [13]. A mathematical development had been given by I.J.Good [26] in 1950 in relation to the weighing of evidence. In the analysis of series over time, Box and Jenkins [9] gave a brief introduction to Bayesian methods with reference to autoregressive models in a work that was primarily non-Bayesian. Zellner [68] was the first to derive a posterior distribution for AR(1) and AR(2) processes showing that, for one-step-ahead forecasts, the predictive distribution is the Student 't'. He also derived a method of sequential forecasting in which the posterior density calculated at one step is used as the prior for the next step, thus using the currently observed values to update the prior knowledge.

The replacement by dynamic linear modelling of a fixed sample of observations based on an assumption of stationarity had been arousing interest for some time with much of the work done using differential equation models or in the frequency domain. However, it soon became apparent that Bayesian methodology was especially suited for forecasting models of this form, and Harrison and Stevens [31] introduced dynamic linear models using conjugate priors and predictive distributions which were closed form and tractable. Sequential analysis was used in this work, implemented by the use of recursive equations for the updating of variables based on the Kalman Filter, details of which are given in Kalman [35] and Kalman and Bucy [36]. Examples of this form of representation are found in Lindley and Smith [49], in Smith [62] and in West, Harrison and Migon [65], who have extended the application to non-linear and non-normal time series and regressions. This approach means that work in the field of forecasting can be effective even for fairly short series or in circumstances where there is little prior information available, for example, as a study of a price series is being initiated. The method of sequential forecasting with updating is a particularly useful aspect of the subjectivist approach.

Subjectivist theory is supported by a number of statistical procedures which include the evaluation of opinions or theories by means of proper scoring rules. For an exposition of these, reference can be made to Blattenberger and Lad [8], Savage [57] and Lindley [47] [48]. The most widely studied is the quadratic difference between expected and actual values. This score was first used in the field of meteorology by Brier [10] and used regularly in empirical work by Lad [44], by Blattenberger and Lad [8] and by Hill [33].

The relation of Bayesian subjectivity to the Rational Expectations approach has been discussed by Swamy et al. [64] who have argued that the different subjective opinions that arise from the agents' state of uncertainty need to be allowed for in modelling and have condemned the confusion between objective and subjective probabilities that often arises in rational expectations models.

1.3.4 The Use of Simulation

Monte Carlo methods of simulation, applied to defence problems during World War II and later extended to the solution of integral equations in the field of physics, were being used by the '50s for the simulation of systems. At first, this use was restricted to the parts of such systems and then extended to the interactions between these parts. As computers became more refined, it became possible by these means to study systems of considerable complexity. These now range from physical systems of interacting particles studied in statistical mechanics, see Chandler [12] to those describing human behaviour. A comprehensive early account of this type of simulation covering voting and other behaviours is to be found in Dutton and Starbuck [16].

When it is a question of examining local interactions in order to draw conclusions about macroeconomic variables, as in a stock market, it is not doubted that influences on a local scale are deterministic. The aggregate structure, however, based on a large number of private motivations, bears a marked similarity to a stochastic process, so that it can be simulated by such a process. The flow of market orders, for instance, may be adequately described by the drawing of buyers and sellers from an appropriate distribution. This technique has been used in applications ranging from that of a normalised Weiner process by Sutera [63] to simulate the effects of those various short-term fluctuations in the ocean currents and atmospheric movements which would by impossible to quantify directly, to the representation by McPhee, Ferguson and Smith [50] of the social discussion and exchange of opinions of a voting population by the generation of probabilities.

Techniques of simulation applicable to human behaviour have been found in various fields. Spatial and spatio-temporal modelling, over different types of lattice and stressing the nearest neighbour concept, Whittle [66] and Besag [5], was easily adapted to human communication and influence in economic and sociological models. More recent developments have taken place in the models of growth and interaction of Renshaw [54] [55], in the use of Markov Chain Monte Carlo algorithms by Besag and Green [6] and others, and in the methods of the neural net and the genetic algorithm. In the Santa Fe Artificial Stock Market, agents who have subjective expectational models recognise and act upon patterns representing states of the market. They have rules for forecasting which can be changed or discarded if proved inaccurate as the learning process progresses. See Arthur et al. [2].

1.4 Aim and Outline of the Thesis

We have seen that the stock market is based on an information processing system where the information itself may take the form of fundamentals, expected or unexpected, of technical rules that depend on movements in the market itself, or of some mixture of both of these. We have also seen that the effective use of information may be influenced by interaction between the traders and by the pressure of market sentiment. Standing in contrast to the philosophy of the EMH, many models have been developed which attempt to explain the volatility that has been observed in asset markets as a breakdown of this efficiency arising either from differences in the information available to the participants in the market or in the processing of this information.

The aim of the current work is to simulate a stock market in which the performance of a set of agents with heterogeneous expectations resulting from the use of different Information Processing Rules can by examined. Each agent has his own subjective model of the market within the same Bayesian form. This model uses the agent's own individual rule for the placing of weights on the information of the current period relative to that of past information. Expectations and the predictions that follow from them are formed on the basis of the current observations of the agents and of their prior opinions. The information represented by the posterior distributions of these agents varies since each individual has drawn different inferences from the data set. Not only do the agents' decisions result in a range of expectations, but, in addition, they differ in the precision of their expectations so that a variety of behaviours ensues. It is the aim of this work to investigate the effects of a range of behaviour patterns arising from the use of these different rules and to compare the market performance of the agents using them. To do this, the weights of each agent are held constant for the duration of their participation in the market.

A model of an order-driven speculative market in a single stock is constructed, with both market and limit orders and no specialists. Traders have a simple portfolio consisting only of the one risky stock and a cash holding and they calculate their demand and supply by maximising a mean-variance utility function. The model is based not so much on traders who are concerned with diversifying their portfolios as on those who are actively seeking out a stock that gives them an opportunity for capital gain. Buy and hold behaviour, when this occurs, results from uncertainty rather than from any long-term planning. So that the different behaviours can more easily be compared, the same initial input is available to all traders, each agent believing that his subjective weightings will give him the advantage over other participants.

Agents' expectations of the current market price take into consideration the last period's price together with any unexpected news. This means that agents, in the first instance, are basing their predictions on a random walk model. But also included in their consideration are several of the approaches of Technical Analysis, in particular, the extrapolation of past trends and a simple buy low, sell high strategy. Market sentiment is also closely observed. Although the formation of expectations according to the constructed market of agents does not conform to Muth's definition of rationality, these agents have nonetheless formed a consistent set of probability values for the relevant events. From this viewpoint, it must be considered to be entirely rational to adopt some of the Chartist approaches, for instance, to buy an overvalued stock if it is confidently expected that a current upward movement will continue and provide an opportunity for profit.

The approach is to simulate the trading activities of a number of agents with sequential updating over successive time periods. It is assumed that all agents update their expectations daily using an adaptation of Bayes' Theorem that allows for the diversity of subjective interpretations. In the simulation this is done by means of recursive equations for the parameters of their forecasts. These equations form the basis of a dynamic system of forecasting which does not need to assume stationarity and which is able to deal with any structural changes as they occur. The information vectors expand at each successive period and the predictive distributions are adjusted. Since conjugate prior distributional forms are being used, the same functional form of mixing function is reproduced at each iteration and each predictive distribution is Student 't'.

Behaviour associated with the use of different Information Processing Rules is studied over both short and long-term horizons as it affects both the shape of the price series and the profits of the agents themselves. Traders are not initially categorised as belonging to any specific philosophical group, but, as the market continues to operate over time, natural groupings begin to form arising from the attributes of the agents themselves. The potential advantages of some of these groupings over others will become apparent as the market evolves.

Chapter 2 explains the design of the simulation. In Chapter 3, the form of the individuals' processing methods is described. First, a function is developed to measure the strength of reaction of individual agents to the most recent information. This function is then embedded into a generalisation of Bayes' Theorem. The decision making of the agents is examined together with their interaction with the market and the strategies adopted by them in their pursuit of capital gain. Chapter 4 analyses the asymptotic patterns of behaviour arising from the agents' differing reactions to the information content of the market. The several distinct types of behaviour that emerge coincide with those described by Lad [41] [42] and provide a basis for further experimentation. The first modification of the trading population is described. Frequently, the more transient patterns of behaviour may be more influential than those emerging in the long term, and these short-term effects, studied over a number of runs of the simulation with different parameter settings, are described in Chapter 5. The insights gained are then used to construct criteria for the development of an evolutionary model of the market which is described in Chapter 6. Chapter 7 presents the results of the application of this model with particular reference to the comparative performance of different types of agents.

Chapter 2

The Simulation of Daily Trading

2.1 Introduction

The aim of this work is to generate observations of the behaviour of simulated agents in controlled circumstances, supplemented by experimentation over an array of control specifications. Starting with the trading decisions of agents based on their own individual strategies and preferences, and studying their interactions with other traders, it is hoped to be able to draw conclusions regarding macroeconomic variables such as volatility and then to see how these variables in turn affect the behaviour of the agents themselves. Moreover, the adequacy of simplistic specification of agent rationality held, for instance, by the theories of "rational expectations" and "efficient markets" can be assessed. The complexity of the detail of this project requires an overview of the structure of the program in order to understand these in context. This is the purpose of this chapter.

Participation in a market is a learning experience. The cumulative effects of the successive updating of information on the patterns of trading and the subsequent revision of opinions that this brings become apparent over time in any market. Simulation is a form of repeated experimentation whereby the same individuals can be placed in a number of different situations in order to assess system characteristics that may not be observable in a single run of history. It enables an experimenter to make repeated reruns in which modifications of the environmental circumstances may provide varying learning opportunities.

A simulated model, the output of which has been compared to a real life series, can then be manipulated experimentally with considerable ease. For example, the response of agents to the crisis situation of a crash can be studied merely by the adjustment of one or two parameters. Simulation is therefore a method that sets out to answer that often asked human question "What if ...?"

There are two parts to this simulation. The first part aims to determine the effects on trading behaviour, both in the long-term and more immediately, of the different information processing rules of individual agents. This is achieved by the running of a main program, each iteration of which represents the activity of a single day's trading. After repetition of this program for the required number of days, the appropriate records are analysed and tested. Where necessary, comparisons are made with other runs set to have differing degrees of interaction between agents or variations in the rate of information flow.

The second part allows a set of agents to develop over time governed by a process of wealth redistribution in which lack of success causes some agents to withdraw while others enter the market. The emphasis here is on the behaviour and composition of the group itself rather than that of individual agents. The main program is now embedded in a sequence of procedures, the purpose of which is to evaluate the success of the agents, to select new entrants and to produce statistics describing the changes in patterns of information processing that are occurring for the experimental set as a whole. This part of the simulation will be described in Chapter 6.

2.2 The Structure of the Simulation

2.2.1 The Storage of Information

The basic unit of the current simulation is an *experimental set* of traders whose behaviour is to be observed while operating in an extended environment, referred to as the *background population.* The agents in these two groups trade equally, follow the same strategies and have the same initial endowments. In the first part of the simulation there is little difference between them because the experimental set did not change. For later work with an evolving experimental set the remainder of the population provides not only a realistic background but a control group with which to compare its financial success. When long-term studies are needed, a regular movement of agents in and out of the background population provides a natural environment.

Details of all agents in the market are held in matrix form as described below. This and other supplementary matrices and vectors provide for both permanent and temporary records for the necessary updating calculations and for the sampling of agents for specific purposes. A set of lists is also used to facilitate the manipulation of the generated data.

A 1000×28 matrix, $MK3$, is set up to store the basic data and to facilitate the calculation of values of the variables used in the simulation. Each row of this matrix represents one agent. Rows are numbered and this figure acts as a convenient identification of the agent. The first 500 entries contain details of agents belonging to the experimental set, while the remaining 500 are used to represent the background population. The columns are used to identify the characteristics of each individual agent. Some character and behavioural properties are fixed, while others which embody current updates of opinions or record current levels of relevant variables change at regular intervals.

Each agent has an initial cash allocation of \$10,000, together with 10,000 shares. Shares are priced initially at \$1 per share and are traded in minimum sized parcels of 100. Odd lots are not being considered. From this point on, the shareholdings will be described in units of 100. The initial allocations are identical over all agents. As the simulation proceeds, total wealth is recorded and regularly updated. This is evaluated as the sum of the agent's cash account and the current value of his shareholding.

The model which is used in this simulation for the formation of expectations and the process of decision making necessitates the specification of a number of parameters, some of which are fixed and some variable. A further set of columns is used for the storage of these. The model which is the justification of these parameters and their initialisation will be covered in the next chapter.

Of the remaining entries in each row, some are used to count the trading activity of the agents, to record the time that has elapsed since an agent entered the market, and to store for comparison the totals for wealth and activity for the periodic examinations of the evolutionary model. A system of $0-1$ coding is used to track those agents who are currently engaged in trading by means of a limit order in order to prevent duplication in the placing of orders. Several other columns in the matrix are reserved to accommodate certain other classifications of agents that arise as a result of the experiments carried out.

Supplementary matrices are generated for the retention of extra information when this is needed. The most important of these is used to keep a record of the daily closing price, the range of prices, the number of transactions made and the volume, being the quantity of shares traded in those transactions. When needed, this provides a picture of the unique configuration of the run being studied. Charts based on this kind of data and similar statistics provide traders in the real world with much of their information about the market results of their expectations and strategies. Similarly, these records form an integral part of this simulation.

Further matrices are used during some runs for storing weekly and monthly statistics and, under some circumstances, even intra-day records were found to be useful. For some parts of the analysis, there is a need for the experimenter to observe long-term changes in the patterns of some variables. For this purpose, detailed records are retained for the individual agents concerned, sometimes over a considerable period of time.

2.2.2 The Use and Organisation of Lists

In a stock market, any orders placed for the purchase or sale of a certain quantity of shares at a predetermined price above or below the current price are passed to a specialist or to a clerk, who stores them in "the book". This book is imitated in this simulation by a set of lists which will now be

		Order Type	
		Profit	Stop Loss
L	L1list	sell ↑	
i	S1list		sell ↓
s	L2list	buy ↓	
t	S2list		buy ↑

Table 2.1: Lists and Corresponding Order Types. Arrows show the direction of price movement needed to activate the orders.

described.

To facilitate the daily search procedure, limit orders are dealt with in four separate lists. The motivation for these is explained by Table 2.1. Orders to sell, triggered by a rising price, are listed in *L1list* and orders to buy, activated by a fall in price, in *L2list*. Stop Loss orders are listed separately since, when placed for precautionary purposes as these are, orders to sell must be activated if prices fall, and to buy if prices rise. Sell orders are recorded in *S1list* and orders to buy in *S2list*.

The format of these lists is now described. Each row contains details of one entry. This consists of the agent's number, his bid or offer price and quantity and provision for the cumulation of these quantities when needed, a code number describing the type of transaction sought and an on–off indicator which is set to '1' when an order is first placed and reset to '0' when it is no longer current. Periodic adjustment of the lists is then easy to carry out.

To a certain extent, the lists just described represent actual records kept, but further lists are formed during the course of the simulation as working lists. These cover activities such as the daily selection of buyers and sellers who may approach the market, *ybid* and *yoffer*; the specification of the probabilities that they will, in fact, attempt to trade, *pbid* and *poffer*; and the storage of activated limit orders, *xbuy* and *xsell*. These latter form a convenient summary of those orders with predetermined prices and quantities that are currently in the hands of the brokers. Finally, a summary list, *Plist*, is used to give a picture of the supply and demand situation for the stock at the opening of each day's trading.

2.2.3 The Codes for Agents' Transaction Strategies

The attitude of the agent as he joins the order flow is influenced by his view of the current price of the stock. Whether he considers it undervalued or overvalued or an accurate reflection of his estimate affects his approach. As a guide to the assessment of this, the artificial bounds, *IL1* and *IL2*, are used, set at the absolute value of 0.025 per share above and below the closing price of the previous day. For example, if the closing price for a parcel of 100 shares is \$100, the next day's bids and offers are considered to be being made with a view to resale or repurchase if they fall outside the values of \$102.5 or \$97.5. A system of coding then describes the following alternatives. An expectation close to the prevailing price suggests a market order to buy or to sell with no immediate follow up. This situation is coded as '0'. Where the prediction of a potential buyer, and with it his bid price, is too low, there is a low probability of a trading partner being found. Where it is high, he will attract sellers easily and make a satisfactory purchase which can then be followed by a resale. These two situations are represented by the code numbers '2' and '4' respectively. The code number '4' indicates that the agent is planning to sell on a rising market. With high expectations, his bid price was high relative to yesterday's closing price and, having made a successful purchase usually below this bid, he is now in a position to consider the placement of a resale order for the same quantity of stock. This agent is therefore confident of making a profit from such trading. Similarly, a seller coded as '3' has predicted a downward movement and wants to sell his stock with the aim of later repurchase at a cheaper price so that he will hold the same amount of shares but be in possession of extra cash.

These codes are set out in Table 2.2.

A further set of code numbers, listed in Table 2.3, are initiated when setting up the lists and are used for the updating of the lists after transactions have been completed or orders abandoned. If a sale takes place an accompanying stop loss order must be cancelled or, alternatively, the original order may have to be deleted when it cannot be fulfilled. Typical entries in *ybid*

Code	Condition	Aim
0	IL1 $\leq$ Order price $\leq$ IL2	Market order. No follow up
1	Asking price > IL2	Bargain hunter, but unlikely to trade
2	Bid price < IL1	As above
3	Asking price < IL1	Sale, followed by repurchase order
4	Bid price > IL2	Purchase, followed by resale order

Table 2.2: Code Numbers for the Identification of Orders

Code No.	Order Type
11	sell, limit
12	sell, stop loss
21	buy, limit
22	buy, stop loss

Table 2.3: Code Numbers for Identification of listed orders

and in *L1list* are shown in Tables 2.4 and 2.5.

Agent No.58 is presenting 31 parcels of shares at the price of \$104.3 per parcel. The code number '4' indicates that his bid is above the current level of IL2. He is therefore planning to resell to make a profit – a decision that is still to be influenced by his last minute assessment of his degree of certainty. The entry in *L1list* shows that he has made a successful purchase and has decided in favour of a limit order placed at the price of \$106.9. The code number '11', which will be transferred at the appropriate time to *xsell*, indicates the nature of this order. If prices fall instead of rising and the agent's stop loss order is thus activated, this limit order can then be deleted. The number '1' in the last column of both entries is used for list adjustment.

No.	Pbuy	Qbuy	code	code
58	104.3	31	4	1

Table 2.4: Details for Agent No.58 entered in *ybid*

No.	Psell	Qsell	spare	code	code
58	106.9	31	0	11	1

Table 2.5: Details for the same agent after a limit order has been entered in *L1list*

2.3 The Trading Day

2.3.1 Time

In this simulation time is measured by activity. Active orders are of two kinds, previously placed limit orders which have been triggered by recent price movements and market orders. The latter, after being drawn from the appropriate distribution of potential buyers or sellers, are recorded by an indicator *tally*. The incrementation of this indicator sets a limit of 100 attempts to trade by market order during any one day. Not all of those who approach the market take part in a successful transaction, since some may be unable to find a suitable trading partner at their specified bid or asking price. These are, however, included in the count. Thus, effective activity from this source is likely to be variable but does not exceed 100 trades. The quantity of shares in each of these trades is, of course, variable.

Limit orders, although not included in the tally, will increase the volume of trade when they are present. The proportion of agents approaching the market who proceed to the placement of further orders varies according to both the individuals' predictions and their states of uncertainty. Also influencing these orders and the times of their anticipated execution is the state of the market at the time of placement. As a result, the size of the limit order lists fluctuates over time. Activation of these orders may, therefore, bring about a considerable increase in volume under some circumstances especially when price movements are large. A sharp downturn, for example, may be accompanied by an accumulation of strategic buying interest while an upward movement stimulates profit taking.

2.3.2 The Global Parameters

The foundation of the simulation is a main program, each iteration of which represents the activity of a single day's trading. This program invokes a number of parameters which simulate the varied aspects of the daily activities of the agents and will be described fully in the following sections.

While individual agents vary in the degree and style of their approach, an important factor in the simulation is the influence of external occurrences or unexpected "news" seen as affecting the future prospects of the stock. The frequency of such events plays a part in the increase or decrease of the volatility of a market which in turn affects the uncertainty experienced by the agents in their decision making. Another influence on the direction of prices arises from the agent's observation of the activities of other agents, both currently and as reflected in past prices. Where such a tendency is marked, it can perpetuate and strengthen existing movements.

To allow for the two factors of external shocks and the propensity of traders to interact with their environment, several parameters are set.

Unexpected News

The frequency of unexpected news is regulated by the variable, n_1. This is a global parameter set over a range of values in $[1, \infty]$. Its aim is to determine the number of days to elapse before the next "shock". The intervals so determined are regular rather than random so that the agents' attributes are easier to isolate and study. It is obvious that extreme values will seldom be used. To imitate a market under very unsettled conditions a value as low as 3 or 4 was tried but, since this means that a high proportion of developments are unanticipated, it is not a realistic scenario over extended periods of time. On the other hand, the very high values which represent an exceptionally stable market can be useful to describe some behaviour characteristics in isolation from external influences.

For the representation of a more realistic market, a level of $n_1 = 25$ has been found successful with settings ranging above or below this when required for comparisons.

The Interaction Parameters

The variable λ_1 sets the degree of trend-following and λ_2 the extent to which agents adjust their bid and asking prices on observation of the current state of the market on the trading day itself. Like n_1, these are both also global parameters. Another interesting approach could be to allow these to vary individually as agents adapt to changes from periods of stability to periods of crisis in the market but, since the main object here is to compare the effects of different behaviours, these parameters are fixed over each run.

λ_1 affects all agents in the market each day as they attempt to predict the next day's movement in price from past prices. λ_2 only affects a portion of the population, those who are actively preparing to trade on a particular day.

Both parameters may range within the interval [0,1]. The values $\lambda_1 = 0.01$ and $\lambda_2 = 0.01$ are used when it is required to represent little or no interaction or to isolate the effects of n_1. To study the characteristics of a crash, high values of both or either one of the parameters can be used. Similarly, the relative magnitudes of each of these parameters can be changed for the purposes of comparison.

Since each of these three parameters obviously influences the effect of the others, and the individual characteristics of the traders as they react also affects the volatility and direction of the price series, a setting of these parameters does not uniquely determine the course of market history. It merely provides a framework that will be continually modified in the course of a run of the simulation by the activities and direct interactions of a number of agents differing in their personal characteristics.

2.3.3 Trading by Limit Order

The first simulated activity of the day is a search of the limit order lists to select those orders that have been activated by the movement of the previous day's prices. At the beginning of a simulation run, this step is omitted until the fifth day, together with the activities associated with the placing of these orders. If forward planning is to be done with any chance of success, this settling in period is needed so that the expectations of the agents may more fully reflect their observations of the market movements.

The procedure for the processing of limit orders is summarised in the following steps. An overall picture is traced diagrammatically in Figure 2.1.

1. **Activation of Limit Orders**

   (a) Observation of the level of the previous day's closing price determines the lists to be searched and the number of orders to be presented on the current day.

   (b) Details are transferred to the appropriate working list, either *xsell* or *xbuy*.

   (c) Working lists are sorted by price to facilitate the matching of those most eager to sell with those most eager to buy.

2. **Fulfilment of Limit Orders**

   (a) A search is made of the working lists in order to locate as many trading partners as possible within these lists themselves with the aim of trading the full quantities specified.

   (b) When a bidder and seller attempt to trade, the first step is to test whether their bid-ask spreads overlap, that is, whether the asking price *Psell* is lower than the bid *Pbuy* so that the conditions of each party are not violated.

   (c) A transaction takes place midway between the two prices quoted for the minimum of the two quantities.

   (d) The appropriate adjustments are then made to the shareholdings and cash accounts of both traders, and accumulating totals such as the level of activity and total number of sales are updated.

   (e) If one partner's requirement is not sufficient to cover the whole amount specified, several trades may need to be made.

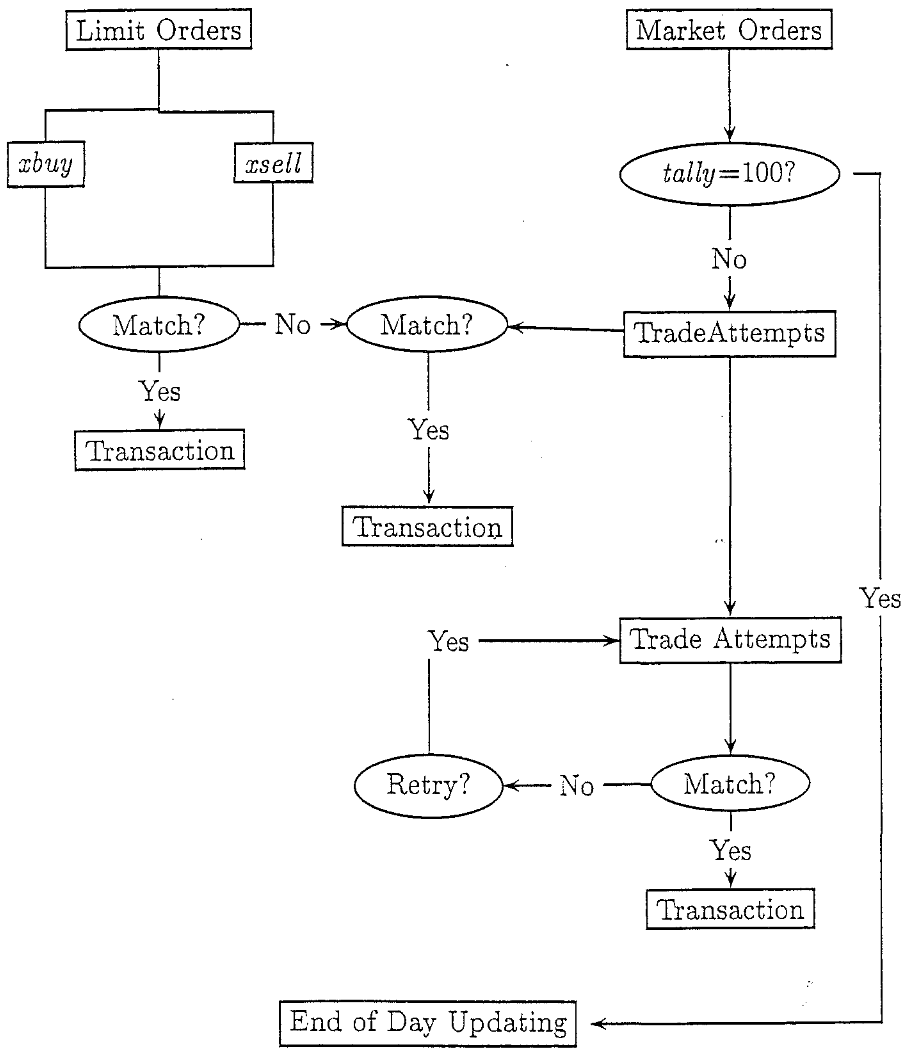

Figure 2.1: Diagrammatic Representation of the Market and Limit Orders - Their Processing and Interaction.

(f) Steps (b) to (e) are continued until either all requirements have been satisfied or there are no more asking prices below the bid prices.

(g) Listed buyers and sellers who are still unsatisfied now attempt to trade with those arriving with market orders.

3. **Adjustment of Limit Order Lists**

(a) Where limit orders have been in place for an extended period of time, agents may review the situation.

(b) Any change of status of limit orders, including the stop-loss orders, is immediately marked and appropriate deletions are made regularly.

2.3.4 The Market Order Flow

The Distribution of Bids and Offers

Although the trader's continuing observation of conditions in the market is recorded each day as this is reflected in his expectations of price and of other measurable quantities, there are other factors influencing his approach to the market on any particular day that are best represented in a simulation by probabilities. The process of selection used here is now described.

1. **Availability**: All agents may attempt to trade by market order if they have no recent limit orders currently in the system. The regular updating of a code in MK3 ensures that unwanted duplications are avoided.

2. **Listing**: The bids and asking prices and the quantities specified for the available agents are listed in *ybid* and *yoffer* together with the necessary coding. The example in Table 2.4 gives an illustration of this.

3. **Formation of the Distributions**: The above lists are sorted by price and the agents ranked according to their eagerness both to bid

for shares and to offer them for sale. Cumulation of the probabilities calculated from these rankings gives the distributions from which agents who actually bid and offer are then chosen. The probability for an agent bidding is proportional to his rank on the list.

4. **Selection**: Agents are selected to approach the market by random drawings from these distributions.

Trading Between Market and Limit Orders

As this flow of orders begins, there may be some entries still remaining in the working lists, either buyers or sellers or both. The next set of procedures steps through each of these lists, entry by entry, attempting to find a trading partner for each from the order flow. Only when this has been completed do those with market orders trade amongst themselves. At this stage, traders with limit orders may need to be satisfied with smaller quantities than originally hoped for. Interaction between limit orders and the flow of market orders for an unmatched seller listed in *xsell* now proceeds in the following steps:

1. **Step 1** Price, quantity and code are recorded for the agent listed who is most eager to sell (the lowest asking price).

2. **Step 2** A buyer is drawn from the distribution of bids.

3. **Step 3** This buyer may adjust his bid at this point according to his assessment of the relevant strength of buyer and seller pressure. Weights for this adjustment are calculated from descriptive statistics about the current state of the market collected in the opening procedure for each day.

4. **Step 4** A transaction is attempted with two possible outcomes:

   (a) A transaction takes place and the various keys are updated accordingly. If the participant with the market order is appropriately coded as planning to resell, any previous order on the lists is replaced or a new one placed. Any stop-loss orders are also dealt with.

(b) Where a trading attempt is not successful, a new buyer is drawn and a new attempt is made on the condition that the new buyer's bid price is above that of the previous one.

5. **Step 5** If no trade has taken place by this time, steps 2, 3 and 4 are repeated once.

6. **Step 6** The entire process is then repeated until there are no more agents in *xsell* who are able to make a match.

7. **Step 7** The process is repeated for *xbuy*.

Trading Among Market Orders

This entails the repetition of steps 2 and 3 above, once for a buyer and again for a seller. Then a transaction is attempted and, if successful, both participants have the opportunity to place limit and stop-loss orders.

If the first attempt is not successful, a simple algorithm is used to extend the circle of possible trading partners through an alternation of bidder and seller replacements. First, another bidder is selected and takes the place of the first provided that a higher price is being presented. If there is still no trade, another seller is selected who attempts a trade if his price is lower than that of the previous seller. The above activities can be summarised as:

(a) Comparison of bid and asking prices among several agents,

(b) Agreement on a sale price and quantity between the participants, and

(c) The opportunity to place limit orders for resale or repurchase of stock that has just been traded.

These activities are repeated until the counter, *'tally'*, has reached the limit of 100 attempts to trade, at which time trading ceases and the final activities of the day take place.

2.3.5 The Closing Activities of the Day

In the interval between the conclusion of the day's trading in a real market and the reopening of the market next day, traders reconsider their expectations and form their plans for the future. This is the time for the acquiring and exchanging of information. The closing price, volume of trade, total number of sales for the day and other details are announced. Screen data, newspaper announcements and other reports are studied, charts drawn up and social discussions engaged in with other agents. Through conversations with other traders, an agent is constantly aware of the relation between his own bid-ask spread and that of the most active and successful traders, whether he belongs to their number or not. It is assumed, for instance, that some may be only too aware that others are acquiring over time an enviable degree of precision in the location of their expected prices which they themselves seem unable to achieve.

In the simulation, a number of records must be kept and updated daily. These records have two main aims. They are needed, firstly, in preparation for the updating of opinions that needs to be done by the agents represented in the model so that their plans may be made for the next day's trading. Secondly, details are stored that will give an overall picture of the situation to the experimenter, enable statistical comparisons to be made and the sampling of specific agents or groups of agents to take place.

These records include the following numbered quantities, the elaboration of which, together with a description of the Information Processing Rules used by each agent will form the subject matter of the next Chapter.

1. Each agent's **expectation** for the next day's price, given his information about the current level of price together with his variance for the next day's price.

2. The data needed for an additional modification of expectations on occasions when **unexpected information** is received.

3. Data to be used for the incorporation of a strategy of **trend-following** by a further modification of expectations.

4. The **prices** at which each agent will offer or bid for stock and the **quantities** he considers optimal to sell or buy at these prices.

5. The current totals of each agent's holding of **assets** in both shares and cash.

6. The total value of each agent's **wealth** at each successive time period, evaluated as the sum of his cash and of his shareholding multiplied by the latest closing price.
7. A measure of the degree of **uncertainty** experienced by traders in the placing of orders.
8. An assessment of the **forecasting performance** of each agent using a Quadratic Scoring Function.

Updating of this set of records is made daily, together with counts of

1. The **number of transactions** completed by each agent.
2. The **number of limit orders** placed.

The periodic sampling of specific agents is also provided for among the final activities of the day, which is then concluded with any necessary reinitialisation and a final adjustment of lists.

2.4 Conclusion

2.4.1 A Qualitative Comparison

In order to test the validity of the methods of simulation used in this model, some of the price series generated were compared to series occurring in the real world. The qualitative similarity between these was encouraging. The representation shown in Figure 2.2 was taken from data in the Christchurch Press for a period ranging from 9th June, 1998, to 29th October, 1998, for closing daily prices in Australian sales of shares of BHP Ltd.

Figure 2.3 presents an example of a series generated by a simulated run for the same number of days. This example was not unusual relative to many checks I made before continuing.

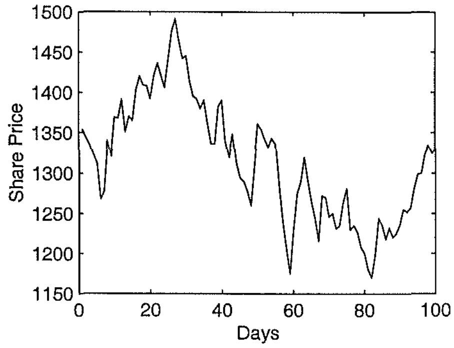

Figure 2.2: BHP Daily Share Price

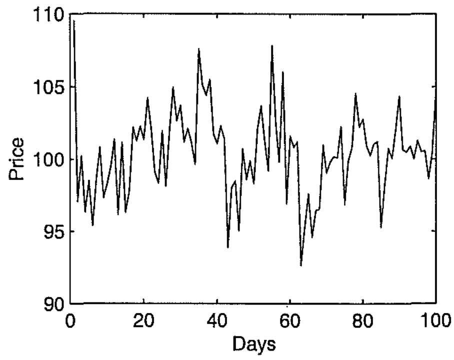

Figure 2.3: Price Series Generated by the Simulation

2.4.2 Specific Objectives of Daily Simulation

The simulation of daily trading is designed to reproduce, at least qualitatively, the overall characteristics of a real world market. However, it has further objectives of a more specific nature.

- To analyse the market as a whole as it influences and is influenced by the participants in that market.
- To illustrate the behaviour patterns represented by the different information processing rules of the agents.
- To examine and compare the potential for successful operation in a speculative market indicated by these processing functions over both long and short time spans.
- To trace the histories of individual agents and to assess their standing at successive points of time.

Chapter 3

The Decision Making of the Agents

3.1 Information and the Agents

3.1.1 The Information Set

In this work, the agent is assumed to enter the market with a background of knowledge and experience together with an individualised rule representing his response to the most recent information. This set of rules is referred to as the *Information Processing Function.* This function was originally designed as a generalisation of Bayes' theorem that could account for an array of learning patterns, particularly obstinacy and timidity in learning. However the parameterised array of learning forms was found to identify distinct but coherent distributional forms that motivate individualised learning behaviour, easily interpretable in a market context. Thus, an array of market participants is quantified by a distribution over the learning parameters describing the agents so that a wide variety of behaviours can be observed.

The agent approaching the market faces an Information Set of generally available price information which he uses to update his opinions so that he may keep pace with the current movements in the market. The first component in this Information Set consists of the regular daily market news, formalised as the observed closing price on each day's trading. This is routinely incorporated into the expectations of agents as they prepare for the following day's trading. The model then allows for a reexamination of past prices from which agents attempt to anticipate the formation of trends. The observation of these movements also becomes part of the daily asessment and applies in varying degrees to all agents.

The simulated market also allows for a quite different component in the Information Set, described in real markets as "unexpected information." This may be conveyed in the form of announcements, political crises or other events, some minor and some of considerable impact. This is information that was not anticipated, but that is likely to bring about a modification of agents' opinions. These events also influence the actions and intentions of all agents. Finally, those who decide to present orders for the next day are subject to still more information as they become aware of the degree of buyer or seller pressure currently affecting the market. This is a factor that, where correctly assessed, may present opportunities for increasing profits.

A set of specific rules used in my simulation to represent the Information Set of the agents can be summarised under four headings:

- the observation of the current day's closing price
- the extrapolation of trends
- the reaction to unexpected news
- the last-minute adjustment of bid and asking prices to perceived demand

Some descriptive background will now be supplied for the last three of these aspects, detailing how they are formalised in the simulation. Overall, agents adjust their daily price expectations as mixtures over the four different assessments.

3.1.2 The Extrapolation of Trends

Short-term trends are a striking feature of speculative price series. Long-term trends do occur, but these are due to changing fundamentals and do not concern us here. Among short-term behaviours of price, the influential ideas of Charles H. Dow described primary, secondary and minor trends.

Dow was founder and first editor of the Wall Street Journal and the Dow Theory was developed by him during the early years of the 20th Century in a number of editorials written for this publication. According to Dow, primary trends may continue for up to twelve months, sometimes longer. Secondary trends, frequently referred to as corrections, are reversals that are of much shorter duration and retrace in the vicinity of 30% - 50% of the previous movement before returning to the primary direction. Within these trends occur a series of fluctuations which are the minor trends. It is these latter in which the short-term traders are mostly interested. It is presumably the alternation between profit-taking on the peaks, causing prices to ease and then to fall, and bargain hunting in the troughs, reversing the decline, that causes them to occur. Each trader's aim is therefore to anticipate, to his own advantage, the movements brought about by the combined strategies of all agents.

In the short run, the extrapolation of past price trends is widespread in speculative markets. This has been described both by Shiller [59] and by Frankel and Froot [23]. There will be agents who will buy on rising prices in the expectation that the rise will continue, thus providing an opportunity for resale in the future. If prices begin to fall, these agents will sell out rather than face a loss. This is frequently expressed in "stop-loss orders" placed to cover a resale order that is aimed at profit-taking.

3.1.3 Unexpected Information

A decision based on an examination of the series of prices, both past and present, is sufficient on most days for the majority of the population. But there is, at intervals, an external source of uncertainty, referred to by the supporters of the Random Walk model of price changes, which insists that prices are unpredictable since the next period's price differs from the current one by a random disturbance only. According to this model, this leaves the current price as the best predictor of that of the next period since any change will be brought about by news that is unexpected.

The Random Walk model can be stated as

$$P_t = P_{t-1} + \varepsilon_t \tag{3.1}$$

where P_t is the price of a share at time t, ε_t is a random disturbance term, $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_{t-s}] = 0$, for $s \neq 0$.

This model asserts that $E[P_t | P_{t-j}, j > 0] = P_{t-1}$, so that all information already known is incorporated in P_{t-1}, including those prior opinions incorporated in the current expectations.

The agent's aim here is to modify his expectations to adapt to this unanticipated eventuality, represented by ε_t, for which no previous preparation has been made and which is consequently accompanied by increased uncertainty.

In the simulated market of this study, there will be a periodic insertion of unexpected information, not in the form of an adjustment to the closing price per se (since every transaction we simulate has a buyer and a seller) but in the form of a common adjustment to every agent's expectation, modulated by the size of each agent's learning parameter. Precise detail shall emerge later.

3.1.4 Adjustment to Demand

When the time comes to present a market order, traders will be attempting to extract further information from their "up to the minute" observations of the current mood of the market. In particular, they are interested in the relative degree of interest in buying and selling, which is seen by traders and others observing the market to reflect the structure of supply and demand affecting it.

As a measure of the buying or selling pressure on any particular day, the simulation uses statistics derived from the currently activated set of limit orders. These limit orders have been placed by agents who are expecting an alternation of peaks and troughs. Following this expectation, the agent sees that the purchase of undervalued stock may mean profits when the next price rise occurs. An alternate, but equally effective, strategy is the sale of overvalued stock and its later repurchase. The final goal, to make profit from the difference in price between the purchase and the sale of stock, is the same in both cases. These strategies complement each other as the participating agents regularly assess the changing state of the market and trade against each other.

Example

An agent who has sold stock on a rise and whose expectation is that prices are due to fall in the near future, is interested in placing an order for repurchase of the stock. In this way, an agent who is watching the market carefully may increase his profits with a minimum of risk. This would be the situation of an agent coded '3' in Table 2.2. At certain critical points, at a suspected peak, for example, after which prices are expected to fall, a large number of agents may make profit-taking sales. Repurchase orders then contribute to a build-up of buying interest at a later date. Sellers note this and take advantage of it to raise their asking prices. New buyers then find they need to be prepared to raise their bids in order to compete.

These last-minute activities of agents about to trade on the day's market will in turn affect prices and influence trends and so feed back into the system itself.

3.2 The Agent in a Speculative Market

3.2.1 The Opinion Structure

In this section, I will begin the analysis of the model of a speculative market used in my simulation. First, I will outline a standard Bayesian learning structure within which the opinions of the agents could be understood to influence their interpretation of the price data. Then more general learning rules will be described and quantified to allow for real and interpretable differences in opinion that motivate the agents to trade among themselves.

Having observed the day's closing price, each agent will assess the next day's price conditional on this observation and on any other previous knowledge he may possess. He may need to modify this assessment after observing evidence from past prices of the formation of a trend or after becoming aware of some announcement which he considers relevant but did not expect. He must then attempt to make a decision about the action he will take. This is not a simple decision, since he remains uncertain of the many and various factors that may still affect the location of tomorrow's price, and consequently the extent of the profit he may make, or even whether his action will result in a profit or in a loss.

No trader knows how tomorrow's price will behave, but each has opinions based on the possibilities as he sees them from his own observations, experience and point of view. On the basis of these opinions, the trader wants to make a statement, as precisely as he can, about the numerical value of the next day's price. A measurement in mathemetical form of personal expectation under uncertain conditions was given by de Finetti [21]. In the context of the operational subjective statistical paradigm, this is known as *prevision.*

When he observes X_t, the closing price at time t, the agent's aim is to state his prevision for a number $(X_{t+1}|\mathbf{X}_t = \mathbf{x}_t, \mathbf{I}_t)$, where $\mathbf{X}_t$ is a vector of past prices and $\mathbf{I}_t$ is the Information Set at time t. He then decides upon his action, that is, whether he will buy, sell or hold stock and, if trading, what quantity he will put forward. By the next day, he has gained extra information which is then incorporated in his prevision for the next day's price, X_{t+2}.

In this work, each agent's distribution for the future price will be expressed as an interpretable modification of a standard conjugate format: a conditional Normal distribution which is a function of the agent's prior expectation, μ, and of the precision for this expectation, π, neither of which are observable quantities. The prior conditional mixture density for the conditional mean expectation, μ, is then in the form of

$$\mu|\pi \sim N(\mu_0, \tau\pi), \tag{3.2}$$

where τ is a scale parameter and μ_0 the agent's expected price. The marginal density for the precision for the expectation is expressed by

$$\pi \sim \Gamma(\alpha, \beta). \tag{3.3}$$

chosen as a natural conjugate prior form.

The individualised modifications to this formulation shall be described shortly via generalised learning rules of information processing. For now let us continue describing the predictive implications of the standard format.

The predictive distribution is thus specified as a Normal-Gamma mixture. The unconditional predictive distribution is obtained by the integration

$$f(X_{t+1}|\mathbf{X}_t,\mathbf{I}_t) = \int_0^{\infty}\int_{-\infty}^{\infty} f(X_{t+1}|\mathbf{X}_t,\mu,\pi,\mathbf{I}_t)f(\mu|\mathbf{X}_t,\pi,\mathbf{I}_t)f(\pi|\mathbf{X}_t,\mathbf{I}_t)d\mu d\pi$$

so that the resulting distribution on the sequence of prices will be a product of conditional Student 't' distributions. For the very first observation X_1,

$$X \sim t(2\alpha, \mu_0, [\tau\alpha/(1+\tau)\beta]), \tag{3.4}$$

with $E(X) = \mu_0, \qquad \alpha > 1/2,$

and

$$Var(X) = \frac{(1+\tau)\beta}{\tau(\alpha-1)}, \qquad \alpha > 1. \tag{3.5}$$

Finally, the bid and offer prices of the traders in the simulation, based on each agent's $Var(X)$, are calculated as the pair, $[\mu - \sigma, \mu + \sigma]$, where $\sigma = \sqrt{Var(X)}$.

Conveniently, the format of the opinion structure described above remains constant through the successive generations of observation values X_1, X_2, X_3, and so on, formalised merely by the updating of values for the values of μ_0, τ, α, and β. Modifications to the standard updating equations are yet to be described.

Note: The assumption behind the standard theory is that the series of observations are distributed conditionally as *i.i.d.* $N(\mu, \pi)$. However this is not really a requirement of the model used in this thesis simulation. For the only structures actually used are the expectation and variance modifications, not the full distributional detail. The observations are, in fact, generated by the market process and each agent sees its distribution as it is influenced by his personal uncertainty. Firstly, the individualised learning rules to be described mean that opinions about the sequence of prices are not exchangeable. Additionally, the normality of opinions is not really crucial to this work, since the family of equations being dealt with pertains to a larger family of distributions even more general, despite the fact that they were originally obtained from Normality theory. This has been shown in the work of Goldstein [25] and more recently in extensions by Lad and Scozzafava [46].

3.2.2 Generalised Learning Rules

The above approach is now generalised in a parameterised form using Bayes' Theorem as the basic structure within which each agent operates by means of his own individual information processing rule. In Bayes' Theorem, the agent's prior information is combined with his observation of the data to form the posterior distribution

$$f(\theta|x) = \frac{L(\theta;x)f(\theta)}{\int_\theta L(\theta;x)f(\theta)d\theta} \tag{3.6}$$

where

$L(\theta;x)$ is the Likelihood Function

$f(\theta)$ is the Prior, and

$\theta = (\mu, \ \ \pi^{-1} = \sigma^2)$.

The changing of opinions according to this formula is widely considered to be the optimal learning method according to principles exposed by Zellner [69] as an efficient markets hypothesis rule. However, the agents depicted in this simulation are not changing their opinions all in the same way as required by Bayes' Theorem using a common distribution specification. A distribution of varying approaches over the trading population is made, so that only in certain special cases is behaviour close to the standard conjugate Bayesian learning encountered.

The model of Lad [41], on which the array of agents' learning adaptations used in this simulation is based, originally developed a generalised form of learning in which Bayes' Rule is seen as a special case within a family of diverse learning rules. This generalised form can be expressed as

$$f_g(\theta|x) = \frac{L(\theta;x)g(\theta,x)f_g(\theta)}{\int_\theta L(\theta;x)g(\theta,x)f_g(\theta)d\theta} \tag{3.7}$$

where

$f_g(\theta)$ is the Prior, and

$g(\theta, x)$ is the individualised Information Processing Function.

The subscript g indicates that this expression is distinct from Bayes' Theorem in that the attitude represented by $g(\theta, x)$ through its relation to the prior always influences the decisions made. The rational learning of the EMH is represented as a special case by $g(\theta, x) = 1$. The heuristic motivation for other forms of $g(\theta, x)$ will be the topic of the next subsection.

Let me mention a final technical matter of the setup. In the work that follows, in order to take advantage of the convenience of the theory of conjugate priors, the precision will be used rather than the variance to identify a distribution's spread. This enables an iterative updating of opinions to take place so that the posterior of one period becomes the prior of the next through standard parametric updating rules.

3.2.3 A Heuristic for the Information Processing Functions

The Representation of the Processing Functions for Expectation

The form that is to be taken by $g(\theta, x)$ must now be specified. To do this a range of possible behaviours that will be representative of stock market participants is considered. Before deriving the individualised rules in precise detail, I shall describe the motivation for the simplest aspect of the rules informally. At one extreme is the type of agent who, on observing the level at which his stock is currently priced, is surprised that it is not at all what he expected it to be. His prior expectation must have been in error, he decides, and acts as if he has forgotten all his previous experiences of the market and hastens to make a change based almost exclusively on this latest information. This is an extreme case, but similar reactions to a lesser degree can be imagined in which decision making tends to be hasty and agents are swayed by the temporary popularity of the latest fad to make precipitate judgments.

At the other extreme, there are those agents who place undue emphasis on their prior learning and on previously formed opinions. These are slower to modify their priors in order to take adequate advantage of new information. Where the former agents are too flexible, these may not be flexible enough. In the more extreme case, the agent makes an assessment of the situation early in his career and chooses to give minimal attention to later information; rather he becomes more and more insistent on initial opinions, with only minor modifications from the earliest of observations. Such an agent is obviously not making an efficient use of the most recent information especially if the market is a rapidly changing one. The majority of agents will, of course, fall between these two extremes. The information processing functions must be so constructed as to embody the range of individual learning behaviours required to describe a representative set of traders in a speculative stock. It is easy to imagine the balance of the standard conjugate learning setup between these two extremes.

In the processing forms used in the present work, $g(\theta, x)$ is set up as a function of the prior. This means that the product $g(\theta, x) f_g(\theta)$ becomes a modification of the prior. Following Lad [41], who described it as an 'impediment function', $g(\theta, x)$ is individualised by a parameter, K, taking positive real values. These values affect the function multiplicatively since, where the prior density is $N(\mu, \sigma^2)$, that of its product with the information processing function is proportional to the density, $N(\mu, K\sigma^2)$. Using the precision, this becomes $N(\mu, \frac{1}{K}\pi)$. It follows that, where $K < 1$, the degree of precision for the product and so for the posterior is greater than it was for the prior, thus exaggerating the importance of the prior in the eyes of the agent with this value of K. Where K is greater than unity, on the other hand, the agent tends more and more to disregard the information contained in the prior and in earlier observations as the data accumulates.

The Processing Functions as Weights

Because of the role the parameter K plays in modifying the prior precision in a sequential learning context, it determines the relative weights placed on the most recent information available to a set of traders who are attempting to make adjustments to their expectations of stock market prices on a daily basis. This can be understood by an examination of the long-term structure it implies for the mean of the posterior density,

$$f_g(\theta|x_1, \ldots, x_n).$$

When posterior computations are computed iteratively, it can be determined that this mean will take the form

$$E_K(\mu|X_1, \ldots, X_n) = \frac{\mu_0 + Kx_1 + \ldots + K^{n-1}x_{n-1} + K^n x_n}{1 + K + \ldots + K^{n-1} + K^n} \tag{3.8}$$

This expression is seen to be a weighted average of the observations and the prior mean. In the limit as n increases, when $K \in (0,1)$, the sum in the denominator becomes $1/(1-K)$, so that the weights on the observations given by the coefficients of the x_j, $(1-K)K^j$, will be declining as the iterations continue. After some time, the more recent observations will carry very little weight for these lower values of K. The rate at which this decline occurs varies, of course, with the magnitude of K. Figure 3.1 shows the long-term limits for weights on a sequence of 50 observations calculated for a range of values of $K < 1$. These observations were made after the trading has settled into a long-term pattern. The bottom line here is that the most recent observations eventually have no weight at all in what is learned.

For an agent with a value of K near to zero, the influence of the most recent observation rapidly decays. When K is closer to unity, especially for values greater than 0.9, later sources of information have relatively more effect on the learner's opinions. Although the movement is still towards zero, this is a much slower process. When $K = 1$, of course, all information is of equal value.

When the lower values of K characterise an agent's forecast, the predicted price is likely to drift in the direction of the initial price assessment. If K is

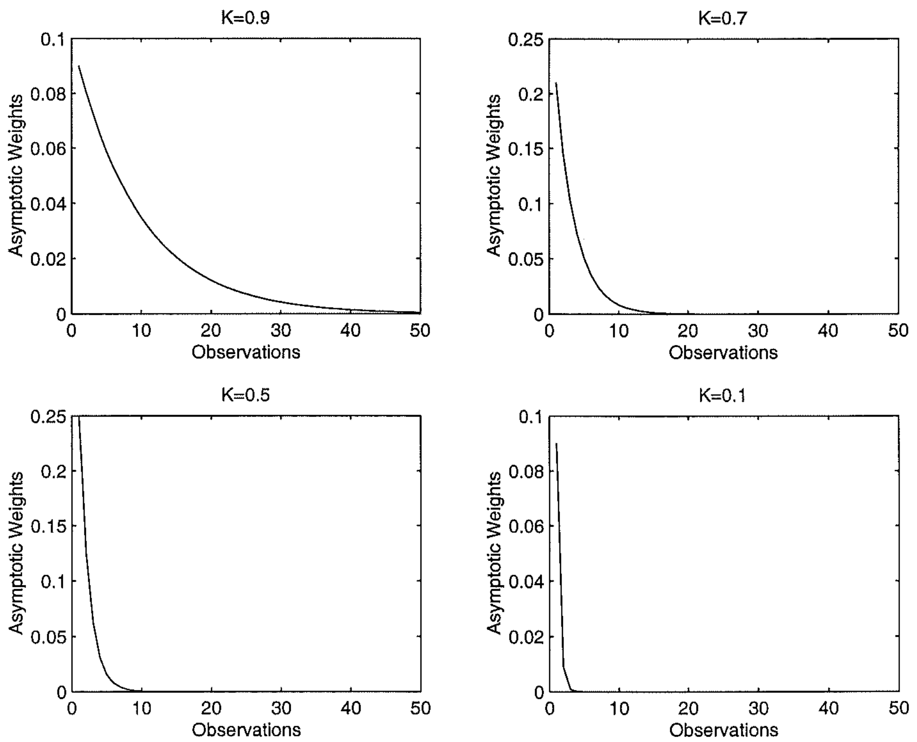

Figure 3.1: Asymptotic weights showing the relative influence of a sequence of 50 observations for individual traders with selected values of $K < 1$.

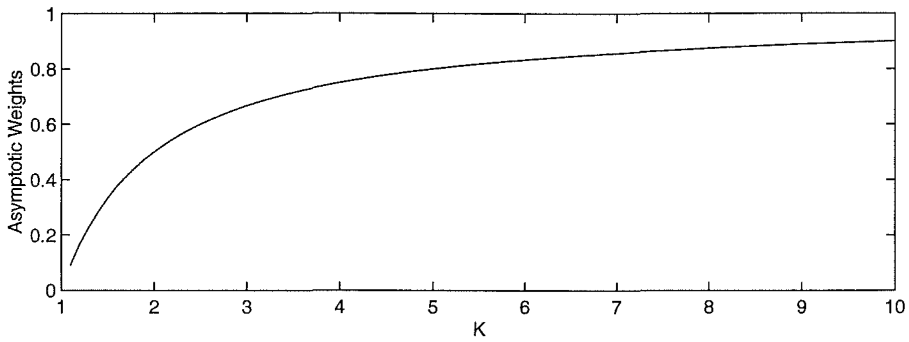

Figure 3.2: Convergence of Weights on X_n for $K \in (0, 10]$. Asymptotically, the weight on the most recent observation converges to $C(K) = (K-1)/K$ which $\rightarrow 1$ as $K \rightarrow \infty$.

close to unity, this may mean only a slow drift in the prediction, whereas, if K is close to zero, there will be little movement away from the early values, implying an almost complete lack of interest in any information from the outside world. One would guess that such an agent would soon be forced to withdraw from the market. The simulations studied in this thesis will guage the extent to which this is the case.

When $K > 1$, on the other hand, the influence of the most recent observations is the most pronounced in a sequence of observations, by a smaller amount when K is very close to 1, but rising with increasing values of K. In this case, the coefficient on x_j in Equation 3.8 now tends toward $C_j(K) = (1-K)K^j/(1-K)^{n+1}$, a sizeable value only when j is near n, for large n. Figure 3.2 shows the long-term convergence levels of weights on the latest period's information for values of K in the interval $(1, 10]$. Of course the weights sum to 1 over all observations for each value of K.

For all values of K exceeding 1, the effects of the initial observations eventually disappear. When K takes on large values, it can be seen that behaviour would be characterised by exaggerated overreactions with almost all of the information considered being taken from a single period. The higher the value of K, the greater is the reaction of the agent to any change in yesterday's price, producing behaviour similar to a random walk with the posterior mean never converging. Details of the convergence characterisation of the varieties of this simple learning rule in an idealised context can be found in Lad [41]. The limiting distribution results found there apply strictly to the context that in fact the sequence of X's are generated as *i.i.d.* $N(\mu_0, \sigma^2)$.

In the market simulation runs of this thesis, the sequence of X's are closing prices from each day's transactions. The sequence has no prespecified distribution. It is the result of trading attempts by the agents, each of whom entertains his own personal distribution which is the basis of his trading offers to buy and to sell. The sequence has no "correct distribution" in itself. Anyone who claims to know one, of course, is welcome to enter the market as a trader. That is what each trader in fact does in the simulation, each in his own way.

To conclude this section, let me remind again that low values of K below 1 emphasise the importance of early observations in the formation of sequential opinions, while higher values of K above 1 emphasise the most recent observation.

The Representation of the Processing Functions for Prevision

A similar modification of standard updating of mixtures over π is also effected by the information processing function $g(\theta, x)$. It was first studied by Lad [42] in the following context. If an agent is currently entertaining a mixture over π in the form of $\pi \sim \Gamma(\alpha, \beta)$, when the chance to update the mixture based on an observation, $X = x$, the updating occurs according to the standard rules of conjugate theory, except that the computation is based on the prior form $\pi \sim \Gamma(\frac{\alpha}{K_\alpha}, \frac{\beta}{K_\beta})$. Similar to the effect of the expectation transformation observed in Equation 3.8, the result of this processing rule is that each observation modifies the posterior expected precision and variance of precision with a different force of weight. After iteratively updating from n observations, the mixture would have the form $\pi \sim \Gamma(\alpha_n, \beta_n)$, where

$$\alpha_n = \alpha K_\alpha^{-n} + (1 - K_\alpha^{-n})/[2(1 - K_\alpha^{-1})], \quad \textit{and} \tag{3.9}$$

$$\beta_n = \beta K_\beta^{-n} + (1/2) \sum_{i=1}^{n} K_\beta^{-(n-i)} x_i^2 \ . \tag{3.10}$$

Using the standard Normal-Gamma equations, $E(\pi|x_1, \ldots, x_n) = \alpha_n/\beta_n$, and $Var(\pi|x_1, \ldots, x_n) = \alpha_n/\beta_n^2$.

Extensive discussion here would deter us from a clear exposition of the expected price patterns, the focus of the present Chapter. Thus, a longer commentary on the precision patterns is deferred until Chapter 4. For now, let us only note that the total sequential learning rule of each agent, in its component of a regular observation of the closing price, is identified by three parameters, K_μ, K_α, and K_β. The parameter K described previously as the learning parameter regarding μ is here and hereafter distinguished as K_μ, as opposed to K_α and K_β which modulate the precision updating. Let us turn to the procedure whereby these parameters are specified for the agents in the market when it first opens.

3.2.4 The Distribution of the Attributes Across Agents

It is now necessary to set up over the population of agents a distribution of the numeric values, K, to represent the learning characteristics which have been described above. It is apparent that when $K = 1$, Equation 3.7 becomes the standard form of Bayes' Theorem shown in Equation 3.6. Applied to the Normal-Gamma mixture distribution, it is appropriate therefore that the distribution of the attributes be centred approximately about 1, with a standard deviation that will give a suitable range. It will need to describe a component of personality present in a population of economic agents in varying degrees. It has been recognised for some time that the Lognormal distribution is well-suited to describe size variables of this kind and has frequently been used in economic applications.

The characterisation of a lognormal distribution that

$$K \sim lognormal(\mu, \sigma^2)$$

will give an array of positive, independent values to denote the magnitude of the agents' predispositions to place weight on the most recent information.

For general purposes, it was found most suitable to use $\mu = 0$ and $\sigma^2 = 0.01$ as values of the parameters of this distribution. Where a wider range of values of K were needed for some experimental situations, $\sigma^2 = 0.04$ was

	$\sigma^2 = 0.01$	$\sigma^2 = 0.04$
Mean	1.005	1.02
Mode	0.99	0.96
Median	1	1
Skewness	0.3	0.61
Kurtosis	3.1623	3.68

Table 3.1: Key Statistics for Two Lognormal Distributions

used with the same value for μ. It is recalled that the above characterisation implies a mean of $e^{\mu+\sigma^2/2}$, a mode of $e^{\mu-\sigma^2}$ and a median of $e^{\mu} = 1$. Some key statistics for both $\sigma^2 = 0.01$ and $\sigma^2 = 0.04$ are set out in Table 3.1.

It has been seen in Section 3.2.1 that, when an agent sets out to predict the next day's price, the distribution on the price is interpreted as a mixture with three different components, μ, α and β, all of which are affected by the agent's information processing rule. Since his attitude in these different aspects of his opinion may vary, a set of three learning parameters, denoted by $(K_\mu, K_\alpha, K_\beta)$ is, therefore, needed for each agent. The weights are not set equal for all three parameters, but each value is determined independently by the same lognormal distribution. Graphs of the initial distributions for each of these parameters with $\sigma^2 = 0.01$ are shown in Figure 3.3.

Detail of the application of the learning rules involving both the expectation and the precision will be specified in the next section. A second paper of Lad [42] on this topic studied the specifics of the learning rule for π in isolation from a rule for μ. My simultaneous treatment of rules for μ and π is novel.

3.3 The Complete Specification of Expectations

3.3.1 Systematic Opinion Change

When the agent prepares to adapt his opinion to the information gained from his observation of X, he does so according to the level of weighting under which he is operating. As we saw in Section 3.2.3, the rule followed is

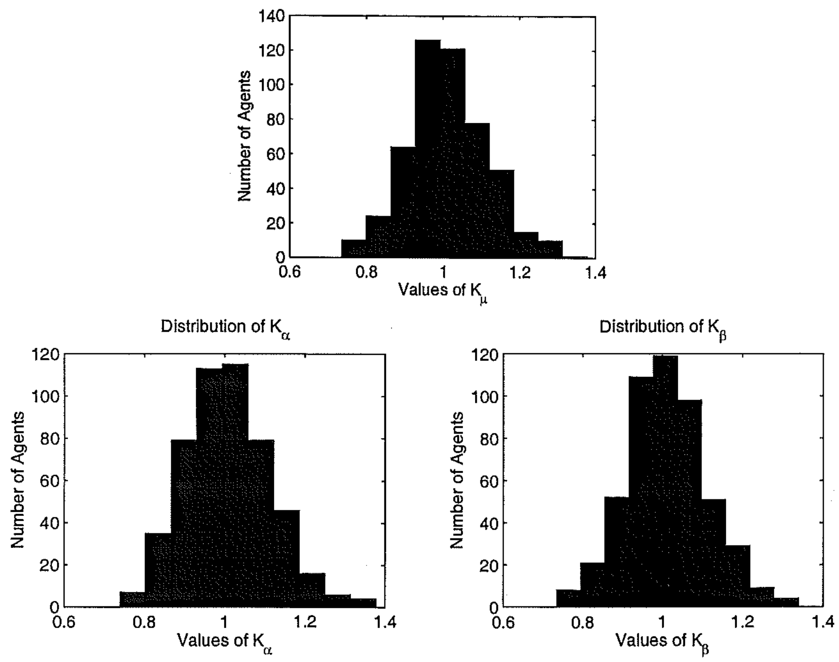

Figure 3.3: Distributions of $(K_\mu, K_\alpha, K_\beta)$

that he does this according to Bayes' Theorem as if the variance-covariance matrix for his prior beliefs is proportional to its actual level. According to this rule, Equation 3.2 becomes

$$\mu|\pi \sim N(\mu_0, \frac{\tau_0}{K_\mu}\pi)$$

while Equation 3.3 can be expressed as

$$\pi \sim \Gamma(\frac{\alpha}{K_\alpha}, \frac{\beta}{K_\beta})$$

The agent's prior density over μ and π will continue to be computed in the form of Normal-Gamma, the product of the conditional density for his expectation and the marginal density for its precision.

This product gives the joint density

$$f_K(\mu, \pi) \propto (\frac{\tau_0}{K_\mu})^{\frac{1}{2}} exp[-\frac{\tau_0}{K_\mu}\pi(\frac{\mu - \mu_0}{2})^2]\pi^{\frac{\alpha}{K_\alpha}-1} exp(-\frac{\beta}{K_\beta}\pi) \tag{3.11}$$

with proportionality constant

$$\frac{1}{\sqrt{2pi}}\frac{(\frac{\beta}{K_\beta})^\alpha}{\Gamma(\frac{\alpha}{K_\alpha})}$$

Multiplication by the likelihood density gives $f_K(\mu, \pi|x)$, the posterior density, which is also distributed as a Normal-Gamma.

This is now expressed in sequential form to illustrate the derivation of the updating equations that will be used in the simulation. It needs to be noted that the use of the sequential method gives a greater emphasis to the weights than would be the case for a single sample, since repeated daily decision-making will magnify the influence of the habitual approach of the agent.

The computation for the conditional inference for $X_{n+1}|X_n = x_n$ gives, for $n = 1$,

$$\begin{aligned}
\mu|\pi, x_1 &\sim N\left(\frac{x_1\pi + \frac{\tau_0}{K_\mu}\pi\mu_0}{\pi + \frac{\tau_0}{K_\mu}\pi}, \pi + \frac{\tau_0}{K_\mu}\pi\right) \\
&= N\left(\frac{x_1 + \frac{\tau_0}{K_\mu}\mu_0}{1 + \frac{\tau_0}{K_\mu}}, \pi(1 + \frac{\tau_0}{K_\mu})\right) \\
&= N\left(\frac{K_\mu x_1 + \tau_0\mu_0}{\tau_0 + K_\mu}, \frac{\pi}{K_\mu}(K_\mu + \tau_0)\right) \qquad (3.12)
\end{aligned}$$

The location parameter for the agent's posterior density is therefore composed of a weighted average of his prior expectation, μ_0, and the most recent observed quantity, x_1.

The update for the next period's expectation is therefore

$$\mu_1 = \frac{K_\mu x_1 + \tau_0\mu_0}{\tau_0 + K_\mu} \qquad (3.13)$$

The weight is proportional to a combination of the precision from the two distributions, that is, the precision for his expectation, $\tau\pi$, and his precision for the new observation, π. With π common to both, it is the size of τ relative to 1 that is important. This means that τ can be updated as

$$\tau_1 = 1 + \frac{\tau_0}{K_\mu} \qquad (3.14)$$

The relevance of Equation 3.9 to the updating of precision will be discussed in the next Chapter.

3.3.2 Modification of μ from Trend Following

Trend following is incorporated in my simulation structure through the daily revision of expectations in which several variables play a part. These are:

- The direction and magnitude of the change in price that has occurred since the last update,
- The relative volume of trading that has accompanied this price change,
- The individual agent's K_μ,
- The global parameter, λ_1.

The change in price is determined by the calculation, $P_t - P_{t-1}$. The volume associated with this change is first ranked by comparison with a stored vector composed of all previously ranked daily values from the beginning of the run. The result is summarised by a variable, δ, taking values between zero and one. A value close to unity indicates that the volume of the previous day was close to the highest ever recorded and a value close to zero that it was the lowest. When the price change is in the same direction as that of the previous day, the significance of the proportion is increased. When this occurs, the final term used for the relative volume is obtained by taking a cumulative sum of ranked volumes over time, embodied in the daily calculation

$$\gamma_t = \delta_t + \delta_{t-1} \quad , \tag{3.15}$$

whereas if the direction of change is different from the preceding day, $\gamma_t = \delta_t$.

The level of each agent's expectation is now adjusted by the following expression

$$\mu_t = \mu_t + \lambda_1 K_\mu \gamma_t (P_t - P_{t-1}). \tag{3.16}$$

The effect is to bias the expectation of each agent in the direction of any trend in recent prices that appears to be forming.

The use of K_μ emphasises the type of reaction characteristic of the individual agent. A trader whose K_μ is well above unity, for instance, will react in a more extreme fashion to an apparently buoyant market sentiment than one for whom this parameter is low. This could provide the former with a greater profit but, on the other hand, expose him to a greater risk of loss. The parameter, λ_1 is used for experimentation since it describes a characteristic of the whole trading population and retains a fixed value for the duration of a complete run. It is used to moderate or accentuate the extent of trend following in a market run, varying from as low as 0.01 when little interaction is desired to as much as 0.65.

3.3.3 Unexpected Modification of μ

The agents have now incorporated in their expectations the information that is available about the price of the stock, both present and past. For most days, this completes the general updating of μ in a different way for each individual; but, in the case of an unexpected event or item of news, a further adjustment is made. The information contained in these "shocks" is assumed to be based on news or rumours concerning the fundamentals of the stock.

According to the Random Walk theory, these shocks occur randomly. In this simulation, for experimental reasons, their occurrence is set at predetermined intervals so that the effects of the agents' Information Processing Functions will be easier to isolate and study. Periods of excessive volatility can be simulated by shorter intervals between shocks and the performance of the agents compared to that of the runs over less eventful periods. Only part of a market's volatility arises from external events, however, since the agents' own reactions are partly responsible for much of the fluctuation. This is particularly evident, for example, when a large number of agents are present with high values of K_μ, resulting in volatility arising from overreaction both to daily price changes and to shocks.

The philosophy underpinning Equation 3.1 is adhered to in this simulation. Accordingly, the agent's adjustment to the news is made by the addition of a term representing the randomness of this event and modified by each agent's value of K_μ. The effect of the unexpected information is closely related to the expectation already determined by the agent. The adjustment used is

$$\mu_t^* = \mu_t(1 + e_t K_\mu)$$

where e_t is an instantiation of ε_t drawn from a normal distribution with mean 0 and a standard deviation of 0.01.

3.4 The Agents' Price-Quantity Pairs

At the end of each day, the agent is aware that tomorrow the price will change in an unknown way. Since he has formulated his opinion about this price as described above, he will make his decision whether to buy, sell or hold stock in terms of his expectation of the value of his portfolio tomorrow.

First, in view of his personal expectations of price, he must determine the prices at which he is prepared to bid for stock or to offer it for sale. Since it is reasonable for the levels of these prices to be related to the agent's $Var(X)$ as specified in Equation 3.5, an arbitrary rule is applied in the simulation that the bid and ask prices are to be set at one standard deviation below and above the expected price. These prices are therefore calculated as a pair, $(p_{buy}, p_{sell}s) = [\mu - \sigma, \mu + \sigma]$, where $\sigma = \sqrt{Var(X)}$. Now that we are well focused on the specific price and quantity analysis, we shall henceforth refer to the unknown X as P. Thus, μ will be replaced by $E(P)$ and $Var(X)$ by $Var(P)$.

Agents then make their decision on quantity by the method of portfolio utility maximisation to be described in the next section.

3.4.1 Portfolio Maximisation and Quantity

Each agent is considered to hold two types of *assets*. One is a risky asset and is denoted here as S, the agent's shareholding in the stock, denominated in units of parcels of 100 shares. The other is in the form of cash or some other risk-free asset, denoted by \$. His portfolio is therefore a simple one, consisting of shares and cash in varying proportions. At any point in time, the agent's *value function* with respect to these two assets can be expressed as $V(S,\$)$. We assume his satisfaction or *utility* is gained from the total value of this portfolio and the aim is assumed to be its maximisation at each approach to the market.

$$V(S,\$) = \$ + SP \tag{3.17}$$

where P is the current price for the minimum trading parcel of 100 shares.

All agents are assumed to be risk-averse, that is, they have a concave *utility function* of monetary value. Such a function can be described by the ascending part of a negative quadratic function: the utility value of the expected return is positive but is modified by the negative effect of the risk taken. Thus an agent will look for the highest expected return that is possible given any particular level of risk. The utility function is therefore an increasing function of the expectation and a decreasing function of the risk, measured by the variance.

The expected utility function for the portfolio value tomorrow can be expressed as a function of the price tomorrow and the number of shares held tomorrow, as

$$\begin{aligned} EU(S,\$) &= aE\left[V(S,\$)\right] - bVar\left[V(S,\$)\right] \\ &= a(\$ + E(P)S) - b(S^2V(P)) \end{aligned} \tag{3.18}$$

where E(P) denotes the expected price and V(P) the variance.

In terms of the price (p_{sell}) and quantity (q_{sell}) that he is prepared to put forward for a proposed sale, the cash holding is adjusted to $\$ + p_{sell}q_{sell}$ and the stock holding adjusts to $S - q_{sell}$. Thus, expected utility of this offer position becomes

$$\begin{aligned} EU(S,\$,p_{sell}) &= aE\left[V(\$ + p_{sell}q_{sell}, S - q_{sell})\right] \\ &- bVar\left[V(\$ + p_{sell}q_{sell}, S - q_{sell})\right] \end{aligned}$$

where p_{sell} is $E(P) + \sigma$ and q_{sell} is the variable with respect to which the function is to be maximised, the quantity to be sold. $S - q_{sell}$ is the quantity of shares remaining after the sale, and $\$ + p_{sell}q_{sell}$ the new cash balance.

Using the value function of Equation 3.17 and the quadratic utility function Equation 3.18 applied to this sell offer then yields

$$\begin{aligned}
EU(S, \$, p_{sell}) &= aE\left[\$ + p_{sell}q_{sell} + (S - q_{sell})P\right] \\
&- bVar\left[\$ + p_{sell}q_{sell} + (S - q_{sell})P\right] \\
&= a\left(\$ + p_{sell}q_{sell} + (S - q_{sell})E(P)\right) \\
&- b(S - q_{sell})^2 V(P)
\end{aligned}$$

Maximising with respect to the quantity to be traded yields a formula for prescribing the quantity to be offered at this selling price:

$$\begin{aligned}
\frac{\partial u}{\partial q_{sell}} &= ap_{sell} - aE(P) + 2b(S - q_{sell})V(P) = 0 \\
&\textit{which implies} \\
2bq_{sell}V(P) &= a(p_{sell} - E(P)) + 2bSV(P); \\
&\textit{that is,} \\
q_{sell} &= a\frac{(p_{sell} - E(P))}{2bV(P)} + S \qquad (3.19)
\end{aligned}$$

which gives the supply function for the agent. This identifies the quantity that he is willing to sell for his offered price.

A similar calculation gives the demand function

$$q_{buy} = a\frac{(E(P) - p_{buy})}{2bV(P)} - S \qquad (3.20)$$

where p_{buy} is $E(P) - \sigma$ and q_{buy} the quantity to be bought.

However, when a purchase is to be made, there is also a budget constraint to be considered. This is based on an arbitrary rule in the simulation that the agent should retain at least one-quarter of his current cash holding, giving the constraint

$$\left(\frac{a(E(P) - p_{buy})}{2bV(P)} - S\right) p_{buy} \leq \frac{3}{4}\$.$$

Thus the programmed buying rule is

$$q_{buy} = min\left(\frac{a(E(P) - p_{buy})}{2bV(P)} - S, \ \frac{0.75\$}{p_{buy}}\right).$$

The corresponding programmed selling rule is

$$q_{sell} = min\left(\frac{a(p_{sell} - E(P))}{2b\sigma^2} + S, \ S\right),$$

since the most that can be sold at any time is the totality of shares held. Short selling is not accommodated in the simulations.

3.4.2 Adjustment to Demand

For those agents participating actively on a particular day by the presentation of market orders, the further adjustment to demand discussed in Section 3.1.4 takes place. This is achieved in the simulation on the basis of:

- the bid-ask spread, $[E(P) - \sigma, E(P) + \sigma]$, of each agent,
- the relative volume of buyers to sellers represented by currently activated listed orders,
- the agent's value of K_μ,
- the interaction parameter, λ_2.

This is done through a modification of $Var(X)$, described by Equation 3.5, represented in the bid-ask spread by its square root, σ. First, relative to the total quantities offered for purchase and for sale, any excess of either buyers or sellers is calculated as a proportion. This is a signed quantity, which will be referred to as Q, positive if there is a greater volume of stock bid and negative if selling volume is heavier. This gives the correct sign for the increase or decrease of each agent's price. To allow for the difference between light and heavy trading, this figure must be seen relative to past trading volumes. For this, a variable, ω, is used, with values in [0,1], calculated in a similar way to δ in Equation 3.15. The bid and ask prices of each agent about to attempt a trade are then modified as follows:

$$P_b = p_{buy} + \lambda_2 \omega Q K_\mu \sigma \quad (3.21)$$
$$P_a = p_{sell} + \lambda_2 \omega Q K_\mu \sigma \quad (3.22)$$

where P_b is the agent's final bid price and P_a his final asking price. The parameter λ_2, like λ_1, is a global parameter that modulates the strength of demand adjustment in any particular market simulation run.

3.5 The Assessment of Precision

3.5.1 The Updating of the Parameters

It is also necessary to define the degree of accuracy with which an agent feels competent to make his prediction of the next day's price. It was seen in Section 3.3.3 that the prior information about the agent's precision is expressed by the Gamma density

$$\pi \sim \Gamma(\frac{\alpha}{K_\alpha}, \frac{\beta}{K_\beta}) \quad ,$$

in the context that "α" and "β" were the appropriate parameters after the previous updating from the previous price, this learning pattern mimics the modulation of balanced learning when both values of $K = 1$, similar to the case for adjustments to μ. We shall note the varieties of asymptotic behaviours these learning patterns allow in Chapter 4.

Ignoring proportionality constants and combining terms in the joint density set out in Equation 3.11, the posterior mixing density for the precision parameter, π, conditional on the observations, is

$$\pi^{(\frac{\alpha}{K_\alpha}+\frac{1}{2})-1} exp[-(\frac{x_1^2}{2} + \frac{\beta}{K_\beta})\pi]$$

which is

$$\Gamma\left(\frac{\alpha}{K_\alpha} + \frac{1}{2}, \quad \frac{x_1^2}{2} + \frac{\beta}{K_\beta}\right)$$

so that the updating equations are

$$\alpha_1 = \frac{\alpha_0}{K_\alpha} + \frac{1}{2} \tag{3.23}$$

and

$$\beta_1 = \frac{\beta_0}{K_\beta} + \frac{x_1^2}{2} \tag{3.24}$$

As regular updating of these parameters takes place, the mean and variance of the precision, α/β and α/β^2, are calculated. In the placing of profit-taking orders in particular, a degree of hesitation could be anticipated on the part of some agents arising from the relative values of this mean and variance. To allow for this uncertainty and to provide a measure for it in my simulation, a rule was specified which will now be described.

3.5.2 A Rule for the Measurement of Doubt

The further uncertainty experienced by some agents in the placing of profit-taking orders is measured by a calculation of the ratio of the standard deviation of the precision to the mean of the precision, which will be referred to as the *doubt factor*,

$$\kappa = \frac{\sqrt{\alpha/\beta^2}}{\alpha/\beta}. \tag{3.25}$$

In other statistical contexts this is commonly called the coefficient of variation.

Experimentation over a number of runs of the simulation led to the partition of the possible values of the doubt factor, κ, shown in Table 3.2 on which the subsequent actions of agents are based. The action denoted as "Decision Uncertain" was designed to mimic the instance that the decision to place an order was based on other factors of the agent's life. In the simulations it is determined by the choice of a random number, to yield a yes or a no on the decision to place an order with probability one-half.

Details of this experimentation are given in Section 5.2. For now, suffice it to say that doubt in the market was deemed naturally to diminish to some extent as market experience increases.

Value	Percentage of Agents		Action
	At 20 Days	At 500 Days	
$\kappa \leq 0.25$	37%	62%	Placement of Limit Order
$0.25 < \kappa \leq 0.5$	50%	27%	Decision Uncertain
$\kappa > 0.5$	13%	11%	No Order

Table 3.2: Values of the Doubt Factor, κ, used for the placement of limit orders.

An Example

An agent who has been strongly influenced by the observation of a rising trend, may find that he has set his bid at a much higher level than the actual price, giving him a competitive advantage in striking a deal over his more conservative co-traders. Assessing the stock as undervalued, his eager entry into the market is likely to result in the purchase of shares at what appears to him to be a bargain price.

However, purchase under these circumstances is regarded as a profit-making move so he will next consider what measures he can take to ensure this end. Such a decision will depend on the degree of certainty that he is experiencing about this forecast now that it is about to be put to the test. If his certainty is high, $\kappa \leq 0.25$, he will place an immediate resell order for the stock he has just bought. A higher value of the doubt factor, however, could bring considerable hesitation or even a suspension of further action altogether with consequent loss of any profit that might have been made. In the simulation, this is effected by placing the limit order only if a test is passed with probability of one-half.

If the agent decides to place an order to resell, the asking price is based on an increase of 5% of the price paid for the stock, with a precautionary stop-loss order set at 2.5% below this price. The quantity does not need to be recalculated since the number of shares purchased is being offered for resale.

An agent who has just sold stock may be in the similar position of having received a better price than he had anticipated. In this case, he may decide to buy into the stock again at the lower price which he had originally predicted and places a limit order accordingly. If he can buy back shares at a lower price, he may be in a favourable position to repeat the profit he has just made. In this case the limit order is to buy back the stock when it drops sufficiently. When the doubt factor, κ is low, he places that order straight away, whereas for intermediate κ he does so only with probability one-half. With a high doubt factor he does not place the limit order at all, because his uncertainty is too great.

3.6 Initialisation

Each agent has an initial cash allocation of \$10,000, together with 100 parcels of shares. Shares are priced initially at \$1 per share and are traded in minimum sized parcels of 100. Odd lots are not being considered. The opening price is accordingly set at \$100.

To begin the iterative process used in the simulation, it is necessary to specify an initial prior density for each agent within the Normal-Gamma family as

$$\mu, \pi \sim N - \Gamma(\mu_0, \tau_0, \alpha_0, \beta_0).$$

The initial price expectation, μ_0, of each agent is set up by drawing randomly from a Normal distribution around 100, with the initial variance of unity. The agent's variance for price, together with the current bid and asking prices for each agent, are routinely calculated at each update together with the accompanying quantities via the K-modified Normal-Gamma mixture equations.

Since

$$\mu | \pi \sim N(\mu_0, \tau_0 \pi),$$

the magnitude of the scale parameter, τ_0, must be determined. Since $\pi(\mu)$ is the inverse of the variance of μ, $V(\mu)$, and $V(\mu)$ is small relative to $V(X|\mu)$, $\pi(\mu)$ is a larger quantity than $\pi(X|\mu)$, that is, $\tau\pi > \pi$. It was therefore decided that the values of τ_0 would be apportioned by making draws from a lognormal distribution, $\tau \sim logN(log10, 0.1)$, allowing τ_0 to vary about 10 among the agents.

A suitable initial value of 0.25 for the mean of the precision, α_0/β_0, gives the values $\alpha_0 = 1$, and $\beta_0 = 4$. These values give the required $\alpha > 1$ from the first update, with a value of 4 for the inverse of $\pi = 1/\sigma^2$, and a variance of the precision of 0.0625. In this way every agent, though being assigned his own personal expected price value to begin the market, is endowed initially with the same value of precision. All succeeding values in the working model are different, of course, modified individually by the agents' K-values.

The iterative procedure being used requires the storage of these and other regularly updated quantities which are individual to each agent. This is done by making routine entries in the columns of the matrix MK3. These quantities stored for each agent are now listed for convenient reference.

1. K_μ, K_α and K_β, which are fixed quantities
2. μ_t and τ_t which change daily
3. α_t and β_t which change daily
4. $EP_t = \mu_t$ and VP_t, given the past day's data
5. α_t/β_t and α_t/β_t^2
6. $\kappa_t = \frac{\sqrt{\alpha_t/\beta_t^2}}{\alpha_t/\beta_t}$
7. The bid and ask prices, p_{buy} and p_{sell}
8. The accompanying quantities, q_{buy} and q_{sell}
9. The cash and shareholdings, \$ and S
10. The total portfolio value
11. The total number of transactions made.

We have now completed the specific computational detail of all processes registered in MK3 that were introduced in Chapter 2. We are ready to begin our analysis of the simulation runs.

Chapter 4

An Initial Analysis of the Population

Preliminary tests were set up in which agents were sampled with the aim of providing an initial overview of the population of traders in order to gain some insight into which of them are likely to survive in the market and the stage of their careers during which they are likely to be most successful. In the first instance, the effects of an Information Processing Rule on the location of the agent's expectation, through K_μ, and its effect on the precision, through K_α and K_β, were examined separately. Special attention was given to the long term effects of these parameters. During this series of tests, interaction between the agents was kept to a minimum and external shocks were avoided, so that the consequences of differing values of these learning parameters could be observed more closely. This was achieved by setting the market process parameters n_1, λ_1, and λ_2 at $500, .01$, and $.01$, respectively. This limits the role of shocks, trend chasing, and demand adjustments quite severely.

4.1 The Location Parameter, K_μ

This section presents the results of an experiment to study the outstanding long-term features of the expectations of individuals with varying values of K_μ. The first step was to study several samples of behaviour patterns,

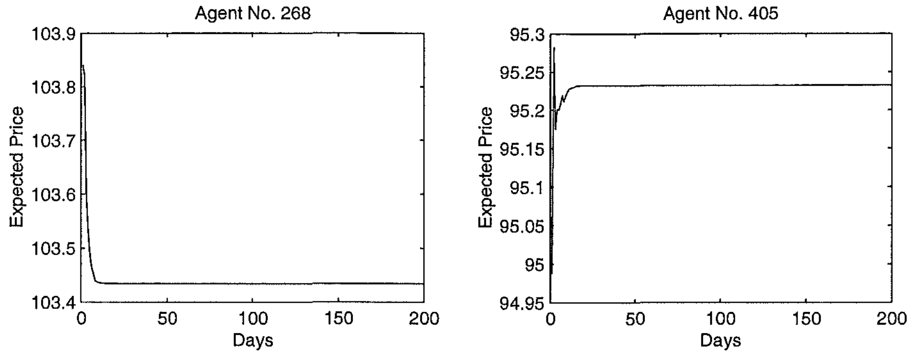

Figure 4.1: Asymptotic Pattern of Expectations for Low Values of K_μ. For Agent No.268, the value of K_μ was 0.5893 and for No.405 it was 0.6342.

stratified according to the magnitude of K_μ. All samples involving $K_\mu < 1$ showed a rapid movement in time towards a steady state expectation for their agents; whereas learning agents characterised by $K_\mu > 1$ exhibited fluttering patterns in their expectations. Nevertheless, the implications of these states for successful forecasting varied considerably.

We recall here Equation 3.8 detailing the conditional expectation for μ by a learner with parameter K:

$$E_K(\mu|X_1,\ldots,X_n) = \frac{\theta_0 + Kx_1 + \ldots + K^{n-1}x_{n-1} + K^n x_n}{1 + K + \ldots + K^{n-1} + K^n} \quad . \tag{4.1}$$

You may also wish to reexamine the accompanying graphical representations in Figures 3.1 and 3.2, which describe the long-term attitudes of agents with different K_μ-parameters to the most recent observations. Keeping in mind the wide spread of attitudes indicated in those illustrations, the asymptotic effects of low, medium and high values of K_μ will be examined separately in the following sections.

4.1.1 Low Values of K_μ

Where K_μ was low, the posterior mean was found to converge comparatively soon to a level that varied from the observed prices with a speed in proportion to the variation of K_μ from unity but modulated by the difference of the first period expectation from 100. The first period μ_0, remember, was selected randomly. The further K_μ was from 1, the stronger the influence of

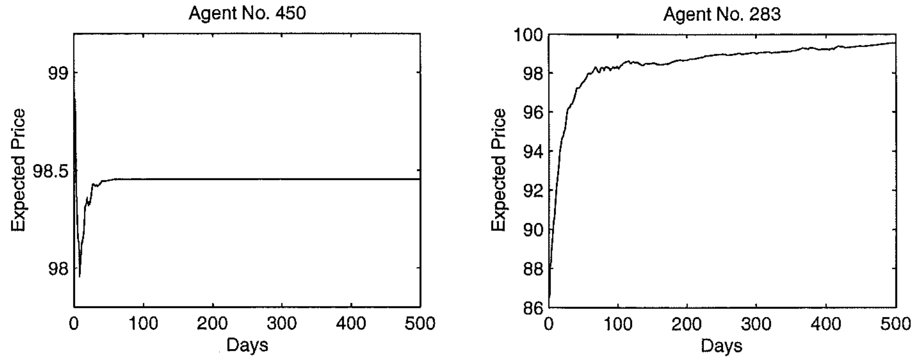

Figure 4.2: Asymptotic Pattern of Expectations for Medium Values of K_μ. For Agent No.450, the value of K_μ was 0.9081, and for No.283 K_μ was 1.004.

the early observations rather than the more recent ones. Of the two agents illustrated in Figure 4.1, the first pattern of expectations, for Agent No. 268, shows convergence to a level of just above \$103.4. The average daily closing price over the first 200 days of this run was calculated at \$99.12. With a K_μ of 0.5893, this agent entered the market with an initial expectation of \$103.97. This has biased the level of the expected value, which remains well above the price series mean. A similar effect is seen in the second graph, but, with a K_μ of 0.6342 and a starting expectation of \$94.9, the error is less and the bias is towards a lower price expectation. In both these examples, weights on the early observations became fixed after comparatively few iterations. The effect produced by these values of K_μ, reflected in poor forecasting, is likely to hamper sucessful trading activity for these agents.

Where $K_\mu < 1$, then, the agent's posterior mean converges, but the value to which it converges is different for each agent depending on the agent's values of K_μ and μ_0. The possibility of extreme convergence values increases as the magnitude of K_μ moves away from 1. Agents with lower value of K_μ are clearly of a less adaptable type.

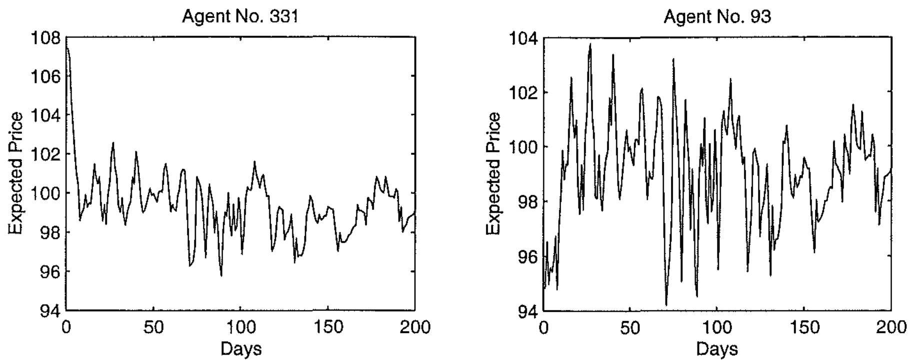

Figure 4.3: Asymptotic Pattern of Expectations for High Values of K_μ. For Agent No.331, K_μ was valued at 1.336, and for No.93 at 1.7127.

4.1.2 Values of K_μ in the Medium Range

When agents' values of K_μ are closer to 1, there is more balance between the influence of recent and more distant information, that is, between the agents' current observations and their priors, resulting in closer approximation of their expectations to the sequential price level.

Agent No. 450, shown in Figure 4.2, taken from the same run as the examples in Figure 4.1, has a higher K_μ of 0.9081 and an initial expectation of \$98.32. Early adjustments took this expectation through a small range of values to become fixed by 50 days at just below \$98.5. At this period, prices were fluctuating about a value close to \$99. This degree of accuracy is, of course, influenced by the weight on the initial guess, although with a K_μ of close to 0.9, this influence is considerably weaker than that seen in the previous two examples. A factor to be considered here is that this agent's accuracy is dependent on the stability of the price series itself which, with the current parameter settings, is considerable. Within a more practical setting, including the strategy of trend-chasing, this agent might have less success, but would nevertheless be able to track the series better than those with a lower value of K_μ.

Agent No. 283 with a K_μ of 1.004 could be classified, within a small margin, as being virtually a Bayesian learner in the Normal-Gamma exchangeable setup. Weights, therefore, are progressively becoming more uniform for each observation of price in this agent's expectation. In this model, the prices are determined by the market activity and a slight upward trend was found to be developing in the present series. The agent's successful tracking of this can be seen over a time period of 500 days in the tendency to an upward slope in the graph of conditional expectations. A sample of agents satisfying the condition $K_\mu \in (0.995, 1.005)$ all showed similar characteristics.

These results roughly correspond to the theoretical analysis by Lad [41] of the asymptotic effects of both the magnitude of the parameter governing the location and of the magnitude and relative positions of the precision parameters. Lad's theoretical results, remember, were based on the assumption of the Normality of the distribution of a series of observations, that is, that the $X_i \sim N(\mu, \sigma^2)$, and show that under this assumption, the expectations of a general learner converge to the generating mean of the series only when $K = 1$. To the contrary, the activities of the market tracked here generate the observations (here market prices) quite differently. The prices are generated by trades among agents each of whom assesses his own distribution for the observable price.

4.1.3 High Values of K_μ

In the case of agents with K_μ values of higher magnitude, the posterior means did not converge but swung back and forth, heavily influenced by the most recent observations, the more so as K_μ lies further from 1. This response also corresponds to Lad's theoretical analysis of normally generated observations, which identifies the features of "wandering means". The plots in Figure 4.3 show two examples. Values for Agent No.331, whose K_μ is 1.336, moved within a range of approximately ± 1.5 about the calculated mean price, whereas for Agent No.93, with a higher K_μ of 1.7127 and therefore higher valuation of the more recent developments, the spread was wider at ± 5.

For this type of agent, as the weightings on lagged observations decreases, the agents' expectations can be described by an autoregressive process. In the most extreme cases, the posterior mean is seen to behave almost as an AR(1) or Markov process, exhibiting sudden changes of direction as the latest news is absorbed and acted upon, with earlier views being virtually neglected.

4.2 The Agent as a Forecaster

4.2.1 The Quadratic Score of the Conditional Expectation

The most basic requirement for successful participation in a speculative market is accurate forecasting. To assess the predictive ability of the agents in our simulation, the *Quadratic Score of the Conditional Expectation*, a proper score, is used. For a detailed description of this score, reference can be made to Buehler [11] or Lad [15]. This score is defined as

$$S_t = [P_t - E(P_t|P_{t-1}, I_{t-1})]^2 \tag{4.2}$$

where

P_t is price at time t, and

I_{t-1} is the information set available to the forecaster at period $t-1$.

This score is a symmetric function of the discrepancy between the current price and the prediction of that price. Since the score is set up as a penalty, a lower score is preferred. The global value for each individual agent is obtained by a cumulative sum of the sequential squared forecast errors. This value can then be compared over different agents, either at the conclusion of a specified period or as an accumulating sequence as required. Cumulative scores can also be ordered so that the performance of the agents can be judged by the percentile in which their scores fall.

Some comparative accumulating scores are shown in Figure 4.4. We note the contrast in the shape of the curves here, with those who score best exhibiting progressively decreasing errors as the number of attempted predictions increases. On the other hand, less efficient forecasters do not achieve this learning effect.

Values of K_μ in the first graph range from 1.0444 to 1.3141, and in the second from 0.7926 to 0.9575. In all cases, the higher value gave the lower

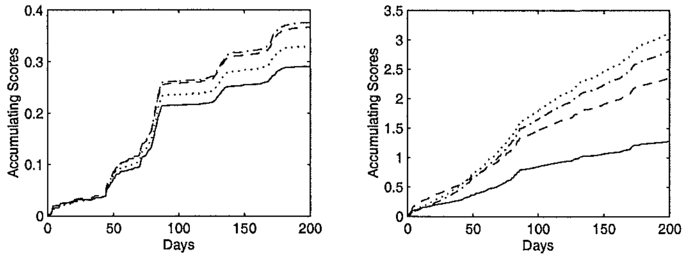

Figure 4.4: Graphs of Daily Cumulative Quadratic Scores. A selection of agents is shown recorded over a period of 200 days. The pertinent values of K_μ are discussed in the text. Notice the different ordinate scales in these two graphs.

score. The scores shown are those for a run of 200 days, with a frequent occurrence of unexpected news. Score values at 200 days ranged from 0.29 to 3.1. It can be seen from the graphs that all forecasts are poor at some of the points where sudden sharp changes in price occur, at which points, as has been seen in Chapter 1, it is notoriously difficult to forecast accurately.

4.2.2 Scoring and K_μ

Over a number of different runs, scores were found to be correlated with the magnitude of K_μ, with a correlation coefficient averaging over all runs around -0.4 and ranging up to -0.55. With this score being designated as a penalty, this implies an association between good forecasting and the higher values of K_μ. This is particularly evident at the extremes of both high and low scoring. Using a number of different records of ordered scores, it was found that less than 10% of the entries in the Upper Quartile of scores, (that is, the poorer scores), have values of K_μ greater than unity, while these values constitute 80% - 100% of the entries in the Lower Quartile. Forecasting is frequently affected by price movements and particular difficulties that occur in a particular run, so that both the level of scores and their range may vary considerably from one run to another. In spite of this, the above results proved to be very consistent. Even the ranking of those agents with the several highest and lowest scores tended to vary very little.

Agents with $K_\mu > 1$ appeared to track the price series with a considerable degree of success. Values of K_μ influence the degree of trend following, but

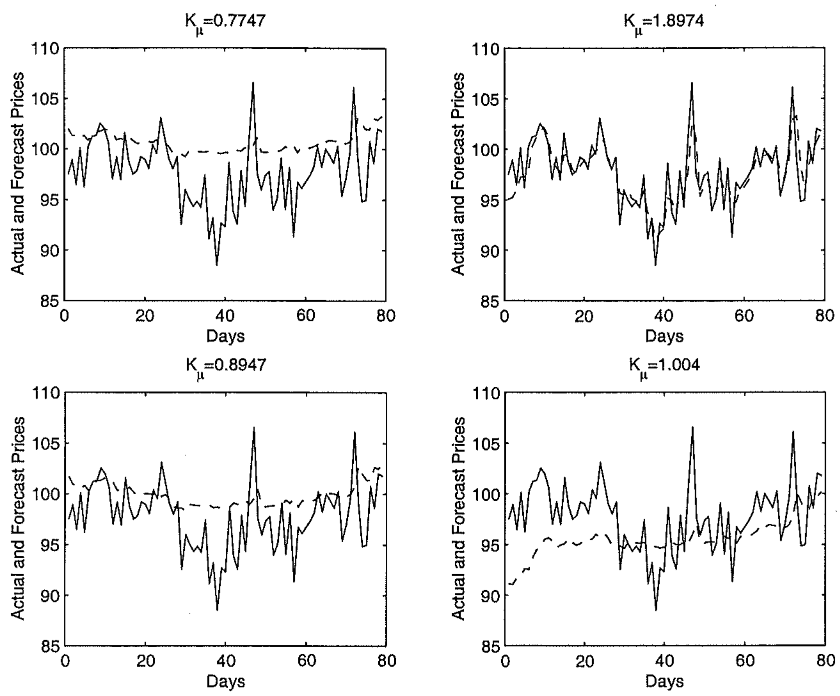

Figure 4.5: A Comparison of Several Forecasts of a Price Series. In each case, the solid line is the actual price series, the dashed line the expected price series. These four examples show agents who have traded successfully despite widely varying forecast success.

we must remember that these values also affect trend formation, especially when daily bid and offer prices are adjusted under buyer or seller pressure. This is the characteristic of markets that the simulation has set out to mimic. Higher values of K_μ give stronger reactions and larger price changes. In turn, successive large price movements in the same direction favour the trading strategies of the speculator.

Graphs in Figure 4.5 demonstrate the wide range of forecasting styles found to be associated with high and low values of K_μ. We recall that, when $K_\mu < 1$, the weighting coefficients of x_n will be declining as the number of iterations increases with the more recent observations carrying very little weight. Price forecasts in such a case are likely to drift in the direction of the initial price assessment. If K_μ is close to unity, this may mean only a slight lag in the prediction with increased dependence on more recent experiences but, as the value falls lower, there may be little movement away from the initial prediction. This is well illustrated in the first graph. The second graph shows the record of the agent with both maximum K_μ recorded, 1.8974, and the best (lowest quadratic) score in the current run. This agent has tracked a volatile series successfully while the agent with a K_μ of 0.7747 has touched it only at scattered points and has missed the downswings completely.

Two intermediate values of K_μ are also illustrated. With K_μ close to 1, these agents score better than those with low K_μ. Where K_μ is 1.004, an initial need of adjustment is seen followed by a tendency to track the mean of the series rather than the peaks and troughs. Agents such as this one represent an approximation to the special case of standard Normal-Gamma learning.

4.2.3 Scoring and the Accumulation of Wealth

There appears to be some relationship, then, between K_μ and the ability to make a good forecast of price insofar as this is measured by the Quadratic Score. The relationship between scores and the attainment of a highly-valued portfolio, however, is not so clear. In the above run, the agent with the highest K_μ, 1.8974, also had the best score and the highest level of wealth, being an increase of 8.5% in 80 days achieved with 9 transactions. However, the agent who could be considered as a standard Normal-Gamma learner, $K_\mu = 1.004$, achieved the fifth highest increase in wealth with 7 transactions. The other two examples, $K_\mu = 0.7747$ and $K_\mu = 0.8947$ also performed well financially with 2 and 4 transactions, respectively. All of these attained an increase in wealth above 3.5%.

It can therefore be seen that financial success is possible for a wide range of values of K_μ seeming to exclude only those of very low magnitude together with a high degree of inaccuracy in the initial assessment of price. The examples quoted were carefully selected to have favourable combinations of precision parameters, but agents with comparable K_μ-values to these do not always succeed. Profits do not always correlate as highly with good scoring as might be expected. Figure 4.6 shows the accumulating scores of

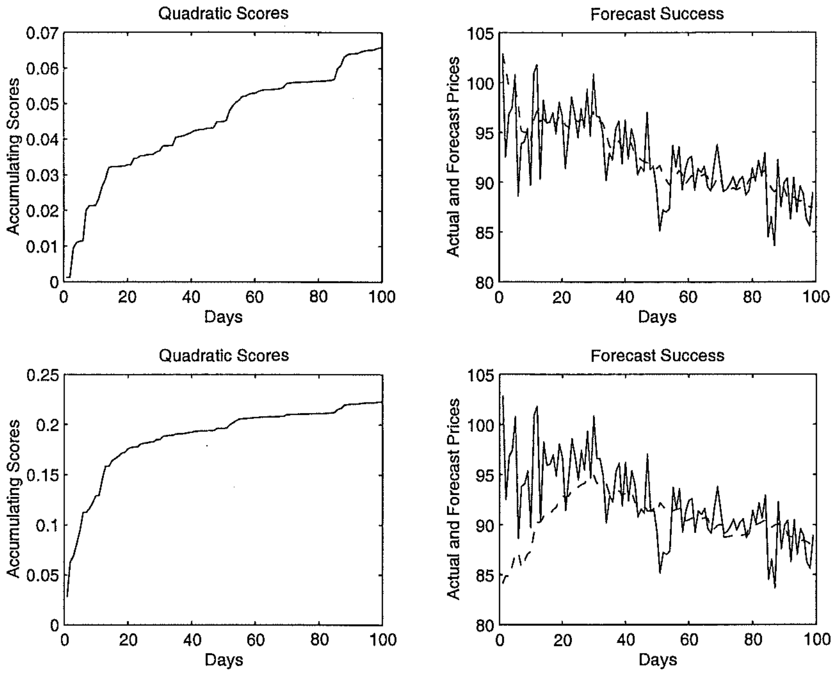

Figure 4.6: Comparison of Cumulative Scores and Forecast Success. The second of the agents shown is highly penalised by initial inaccuracies.

two agents with accompanying graphs showing their success in forecasting. The second of these agents began with the disadvantage of a poor initial estimate of the price at entry into the market. Although the overall trend did follow the direction of his expectations, it took nearly 20 days to adjust with a cumulative score rising in this time to a level well in excess of 0.15. The more successful agent, on the other hand, had adjusted five days earlier at just over 0.03. After 30 days, the predictive accuracy of these two agents did not appear to vary greatly and, since the bid-ask spreads of both became competitive at an early stage, their profits were comparable in spite of the large difference in their cumulative scores.

In these circumstances, the Quadratic Score can be formulated as a tool to make pairwise comparisons between two agents. In cases where some substantial variability at a particular period may be affecting cumulative scores, further insight can be gained by the *incremental score*, a graphical representation of the differences in incremental squared errors between two

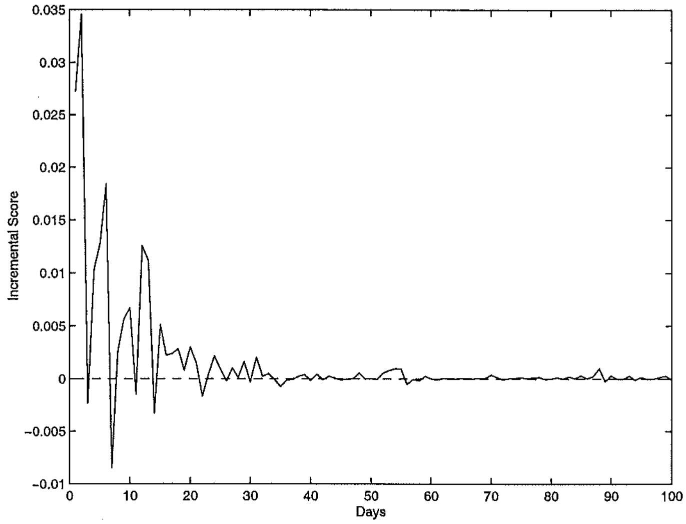

Figure 4.7: Incremental Scores for Two Agents.

forecasters:

$$IncS_t = [P_t - E_2(P_t|P_{t-1}, I_{t-1})]^2 - [P_t - E_1(P_t|P_{t-1}, I_{t-1})]^2.$$

Figure 4.7 displays an example of this incremental score varying about 0. In these comparisons, negative differences indicate a better performance for the second agent.

Incremental scores for the above two agents are shown in Figure 4.7. Negative differences here are few, the only sizeable ones occurring during the period of adjustment when a low starting position has led to a better prediction of some low swings in price. After the obvious superiority of the first agent during the initial period, however, the general impression given by this graph is that the two agents are of approximately equal forecasting ability over most of the period.

In this example, although scores were widely different, forecasting skills were not, and both agents were able to increase their profits. But it has been seen that good forecasts may not always bring financial success. In this market setting, for instance, an agent may not always be able to make a trade when his forecast is good. Other factors may prevent an approach to the market that day or it may be hard to find a trading partner. On the other hand, the difficulty may lie in the agent's precision which may lead to poor pricing or unjustified limit orders or to opportunities being missed if his doubt factor is high.

In the next section, we will concentrate our attention on the patterns that develop with different values of the precision parameters, K_α and K_β, and with their positions relative to each other and to unity.

4.3 Asymptotic Effects of K_α and K_β

4.3.1 Introduction

The Modification of α and β

This part of the work necessitated the comparison of a large number of samples. For each agent sampled, curves for four variables were studied and compared. These were the parameters, α and β, the mean of the precision, α/β, and the variance of the precision, α/β^2.

We recall from the discussion in Section 3.5 that both α and β are modified for each agent by his individual value of K_α and K_β in the series of updates

$$\alpha_{n+1} = \frac{\alpha_n}{K_\alpha} + \frac{1}{2} \tag{4.3}$$

and

$$\beta_{n+1} = \frac{\beta_n}{K_\beta} + \frac{x_{n+1}^2}{2}. \tag{4.4}$$

The values of α_n and β_n achieved by the iterative following of these rules were printed as Equations 3.9, and are repeated here:

$$\alpha_n = \alpha K_\alpha^{-n} + (1 - K_\alpha^{-n})/[2(1 - K_\alpha^{-1})], \quad and \tag{4.5}$$

$$\beta_n = \beta K_\beta^{-n} + (1/2)\sum_{i=1}^{n} K_\beta^{-(n-i)} x_i^2 \ . \tag{4.6}$$

Where $K_\alpha > 1$, then, α_n converges to a real number,

$$\frac{1}{2}\frac{K_\alpha}{K_\alpha - 1}$$

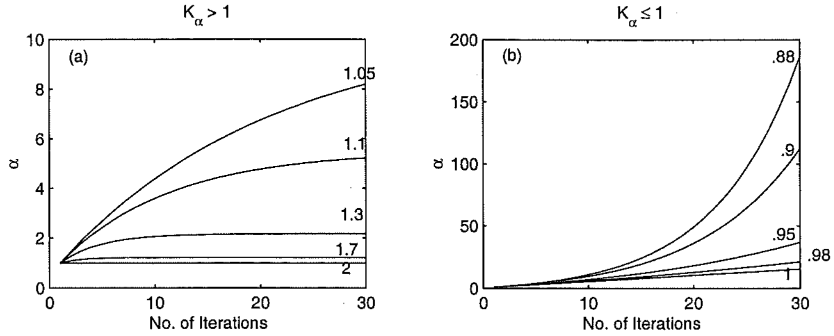

Figure 4.8: Modifications to α for Different Values of K_α

It can be seen from Figure 4.8(a), that a higher K_α gives convergence of α to a lower level, with values of K_α close to unity moving towards a limit that is rapidly rising. At the value of $K_\alpha = 2$, α is constant at unity while, above this value, it would fall below 1, which is not permitted in this model if the moments of the 't' distribution are to be defined.

Where $K_\alpha \leq 1$, the curve for α increases without bound in the long run as shown in Figure 4.8(b). If $K_\alpha = 1$, the curve will be linear, with slope equal to one-half. Where $K_\alpha < 1$, the increase may be very slow at first for values close to 1 but becomes more rapid as the values decrease.

As to the sequential properties of an agent's value of β, in addition to the effect of K_β, they are also influenced by the current value of the price level at the time of the most recent update. Where $K_\beta \leq 1$, and therefore $\beta_n \to \infty$, the effect of this becomes negligible in the long run. For $K_\beta > 1$, however, the convergence of β can only be specified by the mean and variance of a distribution for values of β that are spaced far apart in time. At periods when prices are volatile, the variance of such a distribution is correspondingly volatile and at times may be large.

Configuration Zones

Lad [42] presented a two-parameter characterisation of learning in a Gamma process in which he distinguished analytically eight different configuration zones for the possible asymptotic types of behaviour that might arise with respect to the precision of a Normal process. These zones are reproduced in

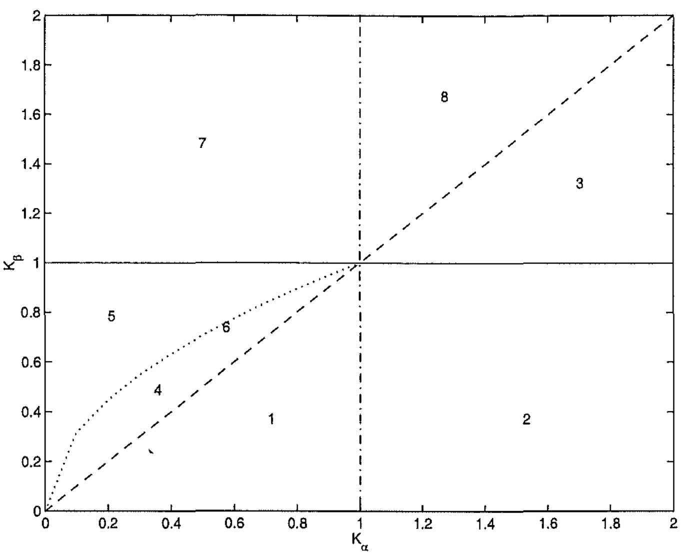

Figure 4.9: Zones of Configuration of (K_α, K_β), adapted from Lad [42]. The zone 6 is defined precisely by the relation $K_\alpha < K_\beta = K_\alpha^{1/2} < 1$. This will be a very important figure to reference through the remainder of this thesis.

Figure 4.9 as they apply to the categories that will now be described. Each possible variation of behaviour is represented by a location in the indicated zones or on their boundaries. For convenience, the numbers assigned to the precision types by Lad are followed in this study.

Information collected over a number of runs, with varying interaction parameters and external stimuli, well illustrated the long-term behavioural characteristics set out by Lad. In some cases, the graphs showed these effects to be evident at a very early stage, in others it was apparent that a longer time span would be needed before the long-term effects would begin to influence the decisions of the agents concerned.

Frequently, the long-term effects of different behaviour patterns were able to be anticipated mathematically by considering the form of the updating equations. In some cases the ratio test

$$\rho = \frac{\alpha_n/\beta_n}{\alpha_{n-1}/\beta_{n-1}} \tag{4.7}$$

can be used to measure the ratios of successive values of the means and variances, and to show whether convergence or divergence is to be expected in these sequences. This test can be used when $K_\alpha < 1$ and $K_\beta < 1$, and Lad [42] has proven that, in a χ^2 data context, when convergence does occur, the limits of ρ are K_β/K_α and K_β^2/K_α respectively. When appropriate, this proved to be a useful tool in order to anticipate the graphical evidence and to confirm the convergence or divergence of the sequences concerned in these simulation runs where the distribution of the trading price sequence is not specified, but up to each agent (and perhaps any analyst who wishes) to assert.

The several categories observed from the simulated data are now described in detail. Section 4.3.2 discusses $K_\beta < K_\alpha$; Section 4.3.3, $K_\alpha < K_\beta, K_\alpha < 1$, and Section 4.3.4 discusses $1 < K_\alpha < K_\beta$.

4.3.2 $K_\beta < K_\alpha$

Immediately noticeable was a group of agents who presented graphs for the mean and variance of the precision that moved quickly towards a value of zero and did not subsequently vary from this. Where both K_α and K_β were below one, it was confirmed that the value of ρ in each of these sequences was a number less than unity, indicating that, in the long run, they would decrease geometrically towards zero. In others, described by $K_\beta < 1 < K_\alpha$, the result is obvious from our knowledge of the fact that K_α converges to a real number for these values of K_α. Typical examples of these two variants are shown in Figures 4.10 and 4.11.

These agents ultimately found themselves making predictions with a complete lack of precision which reflected in the number of exaggerated bid-ask spreads that were being presented by many of them as the simulation progressed. Moreover, since the variance approaches zero, this type of agent becomes absolutely certain that accurate prediction is an impossibility. Two descriptions are involved here, $K_\beta < K_\alpha < 1$, (Type 1), and $K_\beta < 1 < K_\alpha$, (Type 2), but very similar behaviour eventuates. Although, in theory, this belief in complete unpredictability of the share price is an asymptotic effect, the approach towards zero of the posterior mean and variance of the precision tends to occur early, frequently within the first fifty learning experiences.

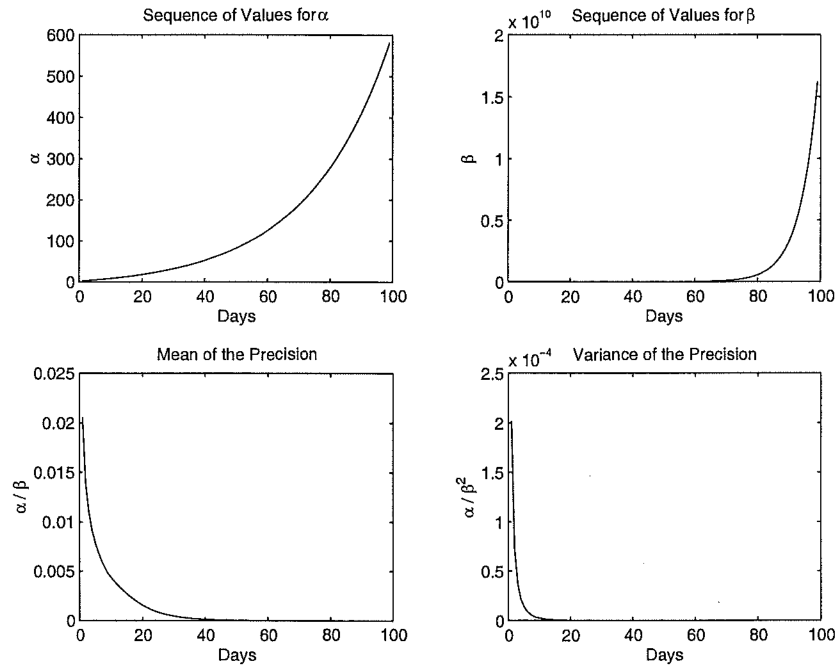

Figure 4.10: This set of graphs shows the asymptotic patterns which evolve where the agent's precision parameters are governed by a Type 1 modification. For this agent, $K_\alpha = 0.9633$ and $K_\beta = 0.8359$.

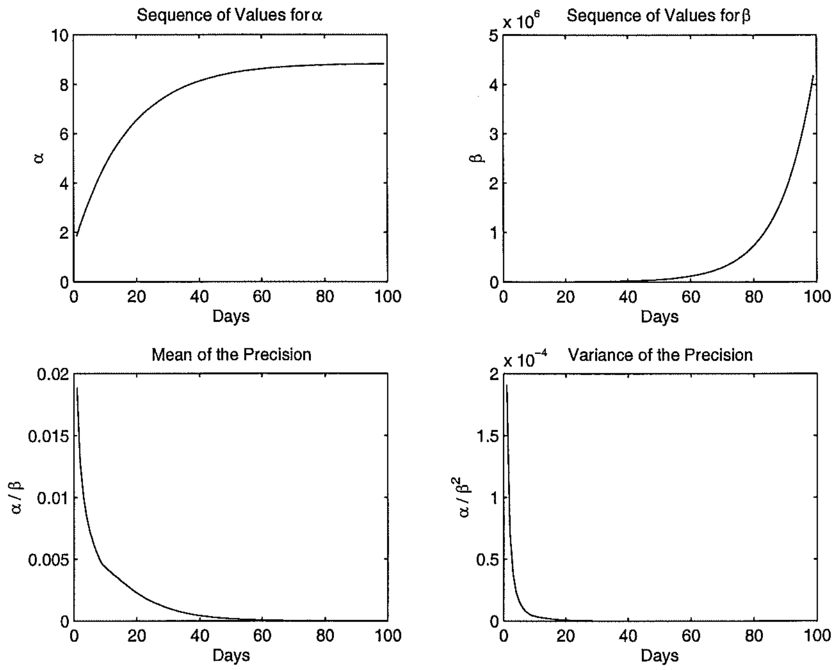

Figure 4.11: Asymptotic patterns for the precision parameters for an agent governed by a Type 2 modification. Here $K_\alpha = 1.0598$ and $K_\beta = 0.9124$.

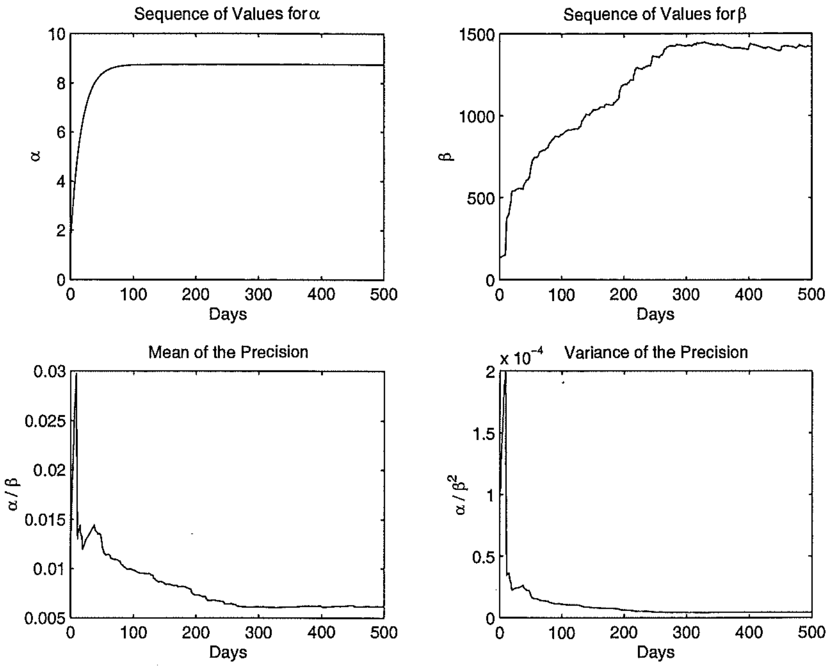

Figure 4.12: Asymptotic patterns for an agent with a Type 3 modification with $K_\alpha = 1.1716$ and $K_\beta = 1.0063$.

We note that both of the above descriptions are relevant in specific situations when $K_\beta < K_\alpha$, so it was appropriate to test other configurations showing this characteristic. This led to the definition of a further type, $1 < K_\beta < K_\alpha$, (Type 3). An experience of a trader of this type is illustrated in Figure 4.12.

For this type, both the mean and variance of the precision remain finite, although the inclusion of the expression $x_{n+1}^2/2$ in the updating equation for β, Equation 4.4, means that there will be a continuing variation in posterior moments for π with changes in price. These agents, therefore, are never certain in their opinions about the volatility of price. Since both K_α and K_β are greater than one, opinions are strongly influenced by the more recent observations, and, because K_β is less than K_α, the asymptotic mean value tends to be low, resulting again in higher standard deviations and so in uncompetitive bid-ask spreads. Most of this type, with similarly low variance of the precision, become fairly certain that prices are quite unpredictable.

There are only a few who may retain hopes of success especially during the more stable periods of trading.

4.3.3 $K_\alpha < K_\beta, \quad K_\alpha < 1$

Since an important part of the activity of a speculator is concerned with the searching out of ways to improve his forecasting skills, it is obvious that the traders most suited to this type of activity will be those who do believe in the basic predictability of prices. One type of curve for the mean of the precision that would give this effect would be one that increased over time, asymptotically increasing without bound. Its inverse then becomes infinitesimally small, providing certainty about the location of the mean price. Whether the location about which he is so confident is that which is being revealed as the current price of the stock will be dependent on the agent's K_μ value.

We now therefore look for ascending curves for α/β. Selecting for the present those agents with both K_α and K_β below or equal to unity the ratio test, described in Equation 4.7, was applied. This revealed a number of sequences for the posterior mean of the precision in which the ratio was found to exceed one, indicating divergence. Thus the forecast precision for these agents becomes more and more accurate as the number of learning experiences increases. However, the variance of the precision for these agents was not so straightforward. In some cases, the ratio of successive variances was found to indicate convergence, ($\rho < 1$), and in others, divergence, ($\rho > 1$), giving asymptotically two different behaviour patterns. In the convergent case, the variance eventually became negligible, leading the trader to a comfortable certainty about his predictions of price near to his expectation level. This situation is illustrated in Figure 4.13. If he is proven wrong at time t, he will be certain he will succeed at time $t+1$, an attitude which, in extreme cases, could leave him open to considerable losses, perhaps even forcing him to withdraw from the market – probably convinced to the last that he had just had an unlucky experience!

In the divergent case, the variance of the precision increases without limit as time passes. How this is likely to affect the certainty of the agent

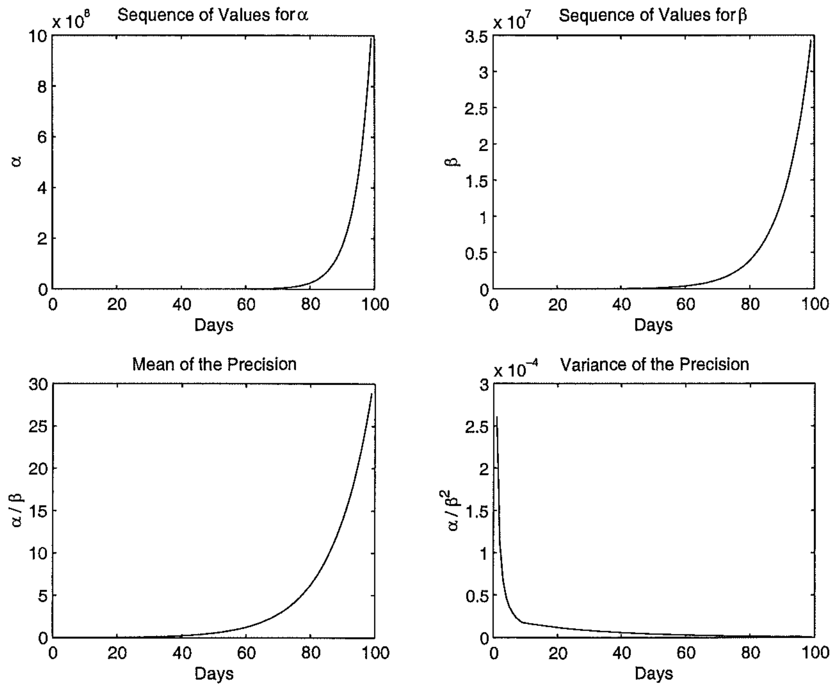

Figure 4.13: Asymptotic patterns for an agent with Type 4 precision parameters. These are $K_\alpha = 0.8227$ and $K_\beta = 0.8921$.

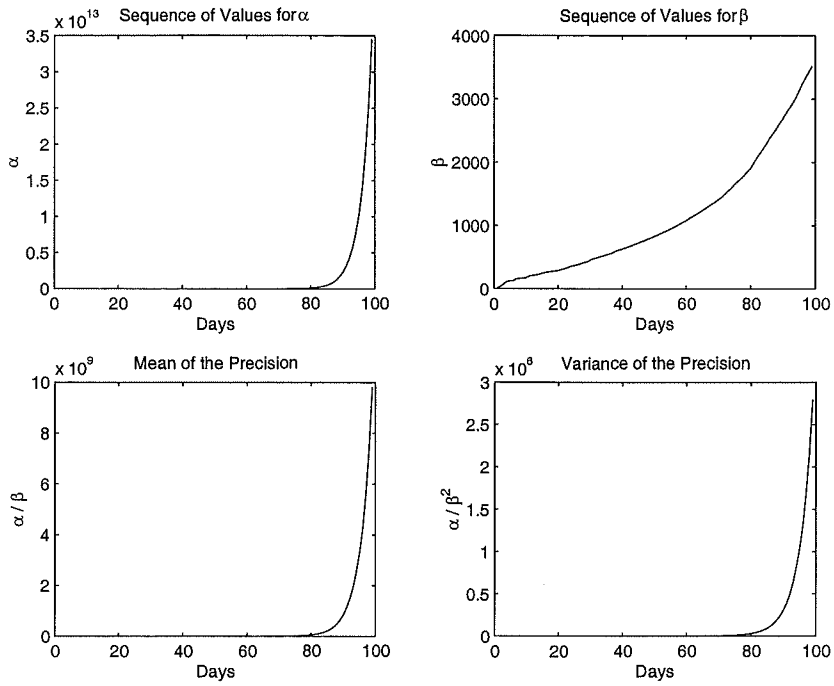

Figure 4.14: Asymptotic patterns for an agent with Type 5 precision parameters. These are $K_\alpha = 0.7387$ and $K_\beta = 0.9754$.

must depend on the comparable increase in the mean of his precision which determines the magnitude of the doubt factor. A study of the example depicted in Figure 4.14 shows how a relatively slow increase in the values of β, together with a more rapid movement of α, has led to extremely precise declarations of expected prices for this agent, but without the certainty exhibited in Figure 4.13.

Since, in the limit, the ratio of successive variances approaches K_β^2/K_α, that point where $K_\beta = \sqrt{K_\alpha}$ is critical to the classification of these agents, which can therefore be described as follows:

Type 4 $K_\alpha < K_\beta < \sqrt{K_\alpha} < 1$, with variance converging to zero,

Type 5 $K_\alpha < \sqrt{K_\alpha} < K_\beta < 1$, with divergent variance,

Type 6 $K_\alpha < K_\beta = \sqrt{K_\alpha} < 1$, a borderline type occurring rarely in which the variance remains finite although somewhat variable.

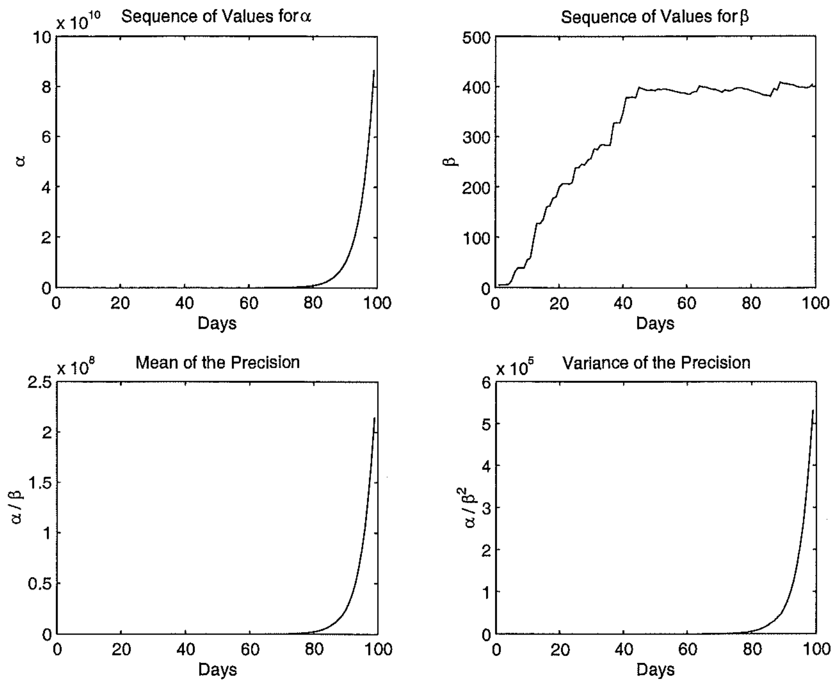

Figure 4.15: Asymptotic patterns for an agent with a Type 7 precision set of parameters, $K_\alpha = 0.7855$ and $K_\beta = 1.0071$.

A similar pattern of behaviour to that of Type 5 is found in another group, which have $K_\beta > 1$, an example of which is given in Figure 4.15

Here β remains finite, varying about a mean value with some variance. The posterior mean of the precision increases without limit as the number of trials becomes large, so that the behaviour of this agent is similar to that of Type 5 but, due to the shape of the β-curve, is subject to a greater degree of uncertainty. If the doubt factor remains low, however, such agents are likely to participate quite successfully in the market.

We can therefore add a further type:

Type 7 $K_\alpha < 1 < K_\beta$, with a variance diverging at a more rapid rate than those of Type 5.

4.3.4 $1 < K_\alpha < K_\beta$

Finally, we examine a group of agents displaying behaviour qualitatively similar in their long-term graphs to those of Type 3 but with less exaggerated characteristics. Figure 4.16 illustrates a typical Type 8 agent. The β-curve here reflects the volatility characteristics of traders with both K_α and K_β above unity with the variation reflected in the mean and variance of the precision, both of which fluctuate widely about a low level. It should be noted that the level of detail depicted here is greater than for the illustration of a Type 3 agent in Figure 4.12, where the values of β reached are very much higher and α converges at a lower level.

The obvious difference between these two types was the level of convergence for the posterior mean of the precision which, due to the transposition in the relative positions of K_α and K_β, occurred on the average at a much higher value for Type 8 than for Type 3. Even so, many of these agents, with standard deviations above the level of their co-traders, were found to be prone to long periods of inactivity. Unlike those of Type 3, these agents tend towards a belief in the predictability of prices. Nevertheless, they show a lack of confidence in the relevance of their declared priors and in the early observations which embody their past experience. Current events frequently take them by surprise, and they will never be certain of their opinions even after many learning experiences.

4.3.5 Conclusion

The distribution of the main types formed by the initial combinations of parameters generated are shown in Table 4.1.

From the results that have been seen as long-term graphs were being generated, it has become increasingly apparent that good forecasts are not likely to be profitable without a sufficient degree of precision. During the various test runs, ANOVA tests were regularly made which showed a highly significant difference between the mean levels of wealth acquired by agents of the different precision types studied here as well as the extent of their trading activity.

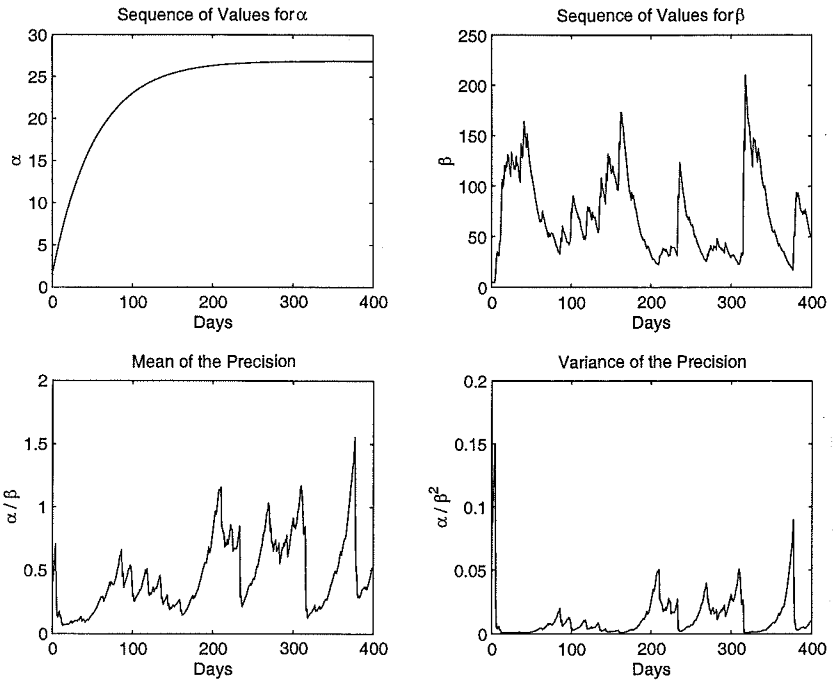

Figure 4.16: Asymptotic Patterns for Precision Parameters Governed by a Type 8 Modification. Here $K_\alpha = 1.0189$ and $K_\beta = 1.0766$.

Type	Percentage
1	11 %
2	24 %
3	14 %
4	4 %
5	5 %
7	23 %
8	13 %

Table 4.1: Percentages of the experimental set belonging to the different precision types for the initial population.

4.4 The First Refinement of the Population

4.4.1 A Problem of Predictability

The preceding results have shown that, while forecasting skill is basic to success in the market, other factors are involved to a considerable degree. Examples were found of individuals who experience difficulties in making successful transactions because of low values of K_μ. On the other hand, others were forecasting adequately but with such a low precision that their forecasts proved useless. And others again were too hesitant to make effective decisions. In some cases, profits were partly due to fortuitous circumstances, as in the first example in Figure 4.5, but consistent success was found to be associated with a favourable combination of the precision parameters. Negative correlation coefficients of close to 0.2 were found regularly between the level of wealth attained and the magnitudes of both standard deviations and of the doubt factor.

In the various runs made with the current set of agents, a problem of predictability soon became apparent. It can be seen that, where the mean of the precision vanishes nearly to zero, as illustrated in Figures 4.10 and 4.11, the agent will have difficulty in making useful forecasts of prices in order to make a capital gain. Since a low precision mean is reflected in wide bid-ask spreads, this type of agent is badly handicapped and, in the long run, spreads become so exaggerated that their position may amount to a withdrawal from the market. This phenomenon was observed widely among those agents with $K_\beta < K_\alpha < 1$ and $K_\beta < 1 < K_\alpha$. These particular configurations resulted in the abstention from trading of many of the population. Figure 4.17 shows the very satisfactory forecasting results of a Type 2 agent in terms of a price expectation that was extremely close to actual prices. In spite of this, this agent made no trades in any of the runs sampled. In Types 1 and 2, it is not only the mean of the precision that goes to zero but also its variance. Agents with these characteristics will eventually become convinced of the intrinsic unpredictability of prices and will therefore experience no enthusiasm for trading in this type of market.

Although this has been classed as an asymptotic effect, it was frequently found to develop at an early stage. An examination of Type 2 agents,

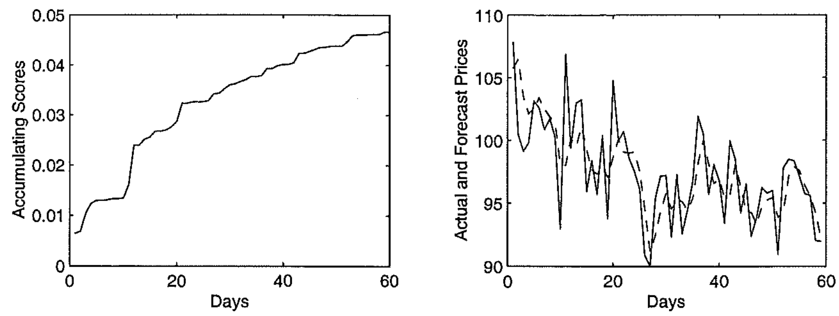

Figure 4.17: This Type 2 agent had a K_μ of 1.5642. The difficulty shown by the sharp increases in the accumulating score at 15 days and again, to a lesser degree, at 20 days was common to all agents and was not the cause of any competitive problems.

numbering 119 in the experimental set, showed that over half had not traded by the 20th day and 32% had made only a single transaction. The mean number of trades for this group was 0.588 compared with a mean of 1.116 for the whole population. By the 40th day, the mean for Type 2 had risen to 0.6 with a maximum for a single individual only of 5 trades. After this point, no further changes occurred. Moreover, the ratio test showed that none were likely.

By comparison, the mean number of trades for the experimental set at 40 days was 1.81 and by 60 days this had risen to 2.96. This was the level at which movement also ceased for agents of Type 1, (53 in all), with a mean of 0.91. Again, the greatest proportion of these traded once or not at all.

It can be seen that the majority of these agents were unable to function successfully after one or two initial transactions. The position of these agents, irrespective of the magnitude of their K_μs, is summarised in Figure 4.18. This result is typical of the various test runs made when these types were included in the population.

A less extreme effect, but one that may be the source of some difficulty for the agents concerned, is found when both K_α and K_β have values greater than unity bringing convergence in the case of both the mean and variance of the precision. The competitiveness of the agents with this characteristic depends, in the long run, on the level at which this convergence takes place,

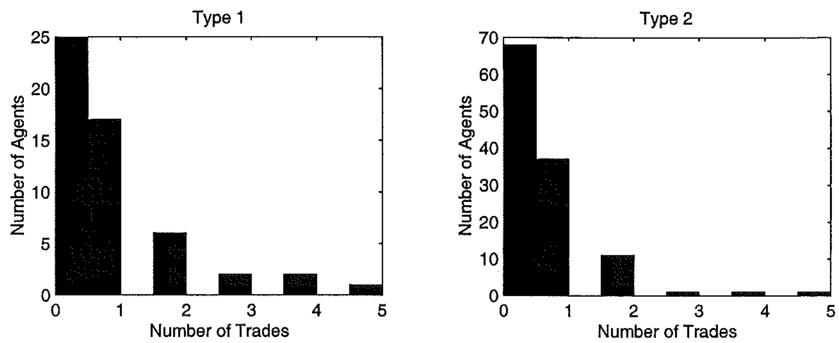

Figure 4.18: Histograms to show the degree of activity of agents belonging to Types 1 and 2. This pattern was established by the 60th day and no further changes took place.

a level that may be considerably lower where K_β is below K_α as in Type 3. Once this level has been reached, these agents may have difficulty competing in the market.

In the same run summarised by the histograms of Figure 4.18, although there was slightly more activity for these agents, the mean number of trades had only reached 0.97 by 80 days, by which time the overall mean had risen to 4.14. Again the majority of agents registered at most one trade. By comparison, it is interesting to note that the maximum number of trades made by any single individual up to this time was 27 – approximately one every three days.

For Type 3 agents, competition is especially difficult when, as a market evolves over an extended period of time, the accumulating experience of the majority of the other agents results in a very low average standard deviation. Figure 4.19, for example, shows the overall distribution of standard deviations for a typical run of 290 days.

An examination of this graph shows that a large percentage of the agents have become very precise in their predictions in just over one year. It has been noted during the simulated runs that the standard deviations of many agents of Type 3 were varying about a level high enough to lead to a great deal of uncertainty. These form approximately the upper 6.8% of values in Figure 4.19. These agents have obviously little chance of competing in such

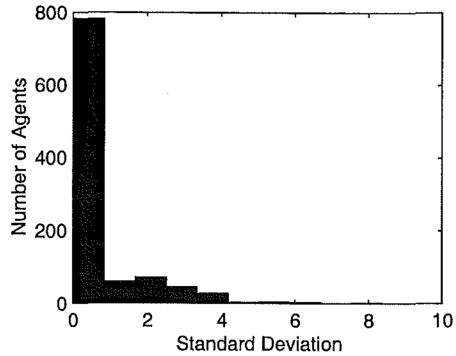

Figure 4.19: Standard Deviations recorded after 290 days.

a market.

A sufficient degree of precision to enable an agent to function adequately was found to occur when K_α was less than K_β so that the 45° line in Figure 4.9 has the effect of dividing the precision types into two main categories – those who believe that prices can be predicted and are prepared to explore the various strategies to accomplish this and those who sooner or later come to believe that they can not.

4.4.2 Selection of the Interested Agents

Having examined the results described in this chapter, it is now the task of the experimenter to determine the composition of a realistic population of agents with which to begin further experimentation.

An immediate problem, that of unpredictability, was highlighted by the last section. This difficulty was found in an extreme form in those agents classed as precision types 1 and 2. As has been seen, few of these have been active in trading even at an early stage, whatever their predictive success may have been. It does not appear possible on the whole for such agents to have useful careers even before they are overcome by their uncertainties. Further, either an application of the ratio test for Type 1 or, alternatively, an algebraic proof using the updating equations for Type 2 shows that the position of such agents will not alter no matter how long they remain in the trading environment. With these characteristics, these types of agents are not suited to short-term trading in a speculative stock. It can be assumed, in fact, that such beliefs have an influence on the overall attitude of these individuals to speculative markets. They will not be fund managers and any private funds held by them are likely to be invested in enterprises they deem as less risky. This assumption was put into effect by the exclusion of Types 1 and 2 from further experiments with this model.

Section 4.2.6 also indicated that many agents with Type 3 characteristics have similar reservations about the predictability of prices although to a lesser degree and a further assessment of these agents was now made.

The essential criterion for any agent with both K_α and K_β in excess of 1 to be successful in a speculative market is based on the proximity of the inverse of the posterior mean to the observed variability of the price series. Once convergence to a level that perpetuates a spread that proves useless for practical purposes has occurred, the agent concerned is not likely to remain in the market.

Lad has shown [42] that, in the case of χ^2 distributed observations, (generated as squares of $N(0, \sigma^2)$ variables), if $K_\alpha > 1$,

$$\alpha_n \rightarrow \frac{1}{2}\frac{K_\alpha}{K_\alpha - 1} \qquad as \qquad n \rightarrow \infty \tag{4.8}$$

while, if $K_\beta > 1$,

$$E[\beta_n] \rightarrow \frac{1}{2}\sigma^2\frac{K_\beta}{K_\beta - 1} \qquad as \qquad n \rightarrow \infty. \tag{4.9}$$

Ignoring the variance in β values, this gives the asymptotic expectation of the inverse of the precision mean

$$E[\frac{\beta}{\alpha}] = \frac{K_\beta}{K_\beta - 1}\frac{K_\alpha - 1}{K_\alpha}\sigma^2 \tag{4.10}$$

$$= c_1\sigma^2 \tag{4.11}$$

where $c_1 < 1$ for $K_\alpha < K_\beta$, (Type 8),

$c_1 = 1$ for $K_\alpha = K_\beta$

and $c_1 > 1$ for $K_\beta < K_\alpha$, (Type 3).

The set of agents of this type discussed in the last section averaged a value of c_1 of 6.8026 with some extremely high values along the boundary of Type 2, that is where K_β was almost unity.

This relationship to the variance of the model is effective in describing agents from the boundary of Type 7, where K_α is close to 1 and c_1 is small in magnitude, through Type 8 and the 45° line to the boundary of Types 2 and 3. It divides agents with both parameters exceeding unity into two groups. Those with $c_1 < 1$ will be interested in making investments on the strength of their predictions, while the other group will be progressively more doubtful of predictability as the magnitude of c_1 increases. A preliminary criterion for Type 3 agents was therefore determined based on c_1. Each agent would be tested and, as long as c_1 did not exceed the value of 1.5, that agent was retained in the model.

A new set of traders was now formed consisting of 1000 individuals. From those remaining after the exclusion of precision types 1 and 2 and of those of Type 3 having $c_1 > 1.5$, a random sample was taken to provide the necessary 500 agents for the experimental set. The background population was now also pretested as well as all agents who might be likely to join the market at any future point of time.

The distributions for K_α and K_β representing this newly formed population are now shown in Figure 4.20. The changes made have not affected the distribution of K_μ. This population was formed by a process of sampling from a $lognormal(0, 0.01)$ distribution. In some of the previous experiments where more extreme values were needed for comparisons, a parallel $lognormal(0, 0.04)$ distribution was used as described in Section 3.2.4. In the same way, a parallel population for the restricted range of types was now set up, illustrated in Figure 4.21.

Naturally, after the changes made, the distributions now vary from those originally set up. Avoidance of the market by agents with $K_\beta < K_\alpha$ has

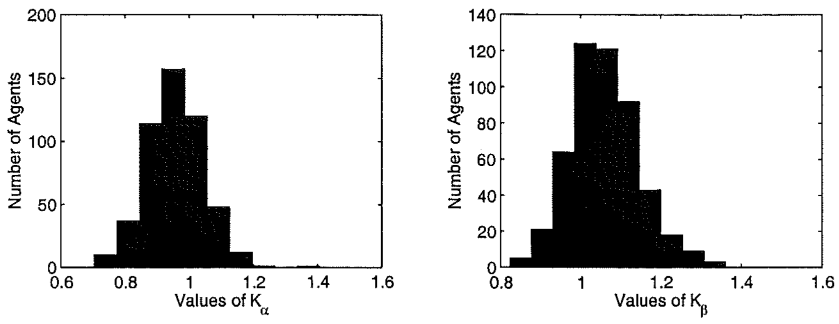

Figure 4.20: Histograms representing the distributions of K_α and K_β for the first refinement for $\sigma = 0.1$. The mean for K_α is now 0.9547 and that for K_β is 1.0607. Medians are 0.9511 and 1.0585, respectively, and standard deviations have fallen to 0.087 and 0.086.

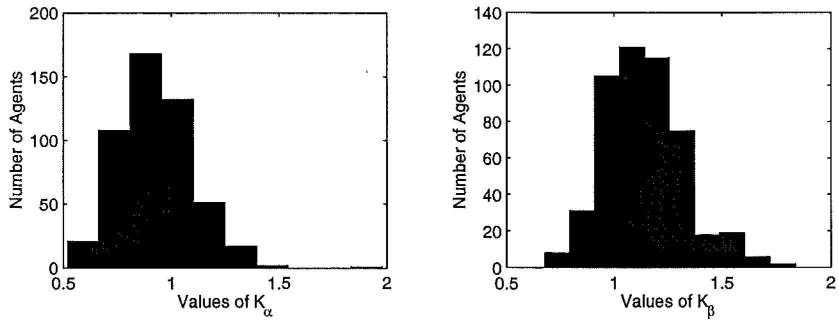

Figure 4.21: Distributions for K_α and K_β for $\sigma = 0.2$. The mean for K_α is now 0.9245 and that for K_β is 1.1386. Medians are 0.9128 and 1.1282, respectively, and standard deviations have fallen to 0.167 and 0.185

Type	Percentage
3	7 %
4	10 %
5	15 %
7	46 %
8	22 %

Table 4.2: Percentages of the precision types after adjustment of the population.

lowered the means and medians of K_α and raised those of K_β. Although the maximum value of K_α is high for both levels of σ, these values are seen to be outliers.

With the exclusion of Types 1 and 2 and the majority of Type 3, the proportions of the types occurring have also changed. The current distribution is given in Table 4.2. These proportions are basically identical whichever lognormal distribution is used to select the K_α and K_β values for the simulation.

The population that has now emerged consists of traders who are prepared to test their forecasting skills in the stock market believing that it is possible to trade in such a way that financial gains will be the reward of a skilful study of market conditions. If a few of these agents (Type 3) do have some doubts, these have at least made an initial decision to try their luck. The extent to which they ultimately decide to act upon their forecasts depends on the variance of the precision and will be influenced by the degree of successful activity engaged in.

Chapter 5

Behaviour in the Short-term

The asymptotic analyses of K_μ and of the different combinations within the subset (K_α, K_β) has given a useful overall picture of a number of different types of behaviour. While these tests were being made, the presence of individuals who would not, in more realistic circumstances, be part of such a trading situation, made any adequate overall study of certain short-term aspects difficult. Of greatest importance among these were the questions of the uncertainty which is manifested in the two aspects of the precision, the standard deviation and the doubt factor. Also, with so many either abstaining altogether from trading activity or attempting to place orders at unrealistic levels, it was impossible to test the real profit-making capability of the other agents in a more normal environment. With the modifications made to the trading population described in the last chapter, however, such questions could be addressed.

If the dynamic evolution of this model is to be put into effect, more detail is needed in understanding the various transient states through which the agents are likely to pass before the structure of their behaviours become regular. Short-term predictions are important in speculative trading and early patterns of profit-making may be vital to the continuation of trading. This Chapter reports on the short-term behaviour of the agents. First, some of its problems are discussed followed by a survey of short-term financial performance.

5.1 The Standard Deviation Problem Revisited

Since every bid and asking price presented in the market is influenced by the unique precision parameters of the presenting agent, the composition of the variance for the price from which these prices are calculated is recalled here in detail. This variance is given by

$$V(X_{t+1}|\mathbf{X_t}, \mathbf{I_t}) = \frac{(1+\tau_t)\beta_t}{K_\beta} \frac{K_\alpha}{\tau_t(\alpha_t - K_\alpha)}$$

The importance of the standard deviations for the next period's price, on which the bid-ask spreads of the agents depend, lies less in their absolute magnitude than in their size relative to that of others in the simulated market at the time when orders are presented. When the precision of the expectations is substantially less for some groups of traders than for the majority, those with the wider spreads are at a disadvantage.

A wide spread about a good forecast of price leads to a bid price too low to attract sellers or to an uncompetetive selling price. On the other hand, if the spread is about a forecast that is located far too high, a transaction may be made. But now a limit order placed in the hope of making a profit is likely to be left unsatisfied unless the price series were to rise sharply. This situation where one error cancels the other is a possible but highly unreliable source of profit. A more likely scenario in the case of wide spreads is the inactivity they generate. When criteria are being established for survival in a longer-term market, it is important to know whether periods of inactivity are to be expected and, if so, to what extent they should be allowed for.

5.1.1 Some Special Cases

The analysis in Chapter 4 showed that, in the case of a large number of agents, the posterior mean of the precision explodes in the long-run. This occurs in 70% of the trading population as it is currently constituted. This situation would seem to indicate few problems in the setting of the bid-ask spreads of these agents. As long as they possess adequate forecasting skills and have a low doubt factor, such agents should be able to function well as soon as they reach a satisfactory level of precision. This, in many cases, occurs early. Figure 5.2 shows two typical representatives of these

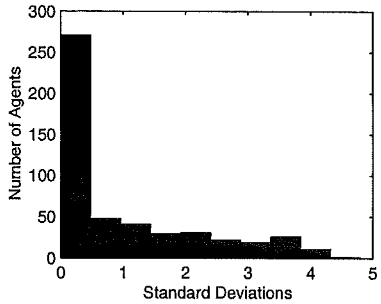

Figure 5.1: Distribution of Standard Deviations of the Experimental set at 200 Days

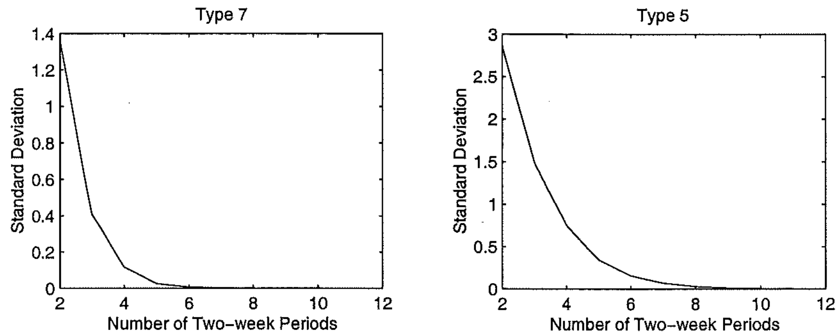

Figure 5.2: Recorded Standard Deviations for Two Agents with High Precision.

types. When compared with the overall distribution of standard deviations, illustrated in Figure 5.1, it can be seen that neither of these agents is being hindered by lack of precision. Records show that both are reasonably active in the market and both have increased their wealth.

The most numerous type in the present population is Type 7, $K_\alpha < 1 < K_\beta$, accounting for 46% of the total, the majority of whom behave in the above manner. There are, however, approximately 10% of this group who face the market at successive time periods with a pattern of standard deviations that, in the short term at least, contradicts what appears to be the norm for this type. This is illustrated by Table 5.1 which shows a selection of standard deviations recorded for this type at successive time intervals of two weeks over a period of 60 days counted from the beginning of a run. The final column gives figures for the same agents taken at 200

Standard Deviations

10 Days	20 Days	30 Days	40 Days	50 Days	60 Days	200 Days
2.5581	1.5972	1.4143	1.5284	0.9432	1.1934	0.4197
2.7171	1.8605	1.6559	1.7306	1.1856	1.3808	0.5210
2.9861	2.4030	2.2637	2.3068	1.8949	1.9898	0.9784
2.9790	2.2827	2.1009	2.1476	1.6573	1.7883	0.7796
1.1910	0.3229	0.0965	0.0311	0.0068	0.0023	0.0010
2.0316	1.1096	0.7163	0.5056	0.2567	0.1861	0.0001

Table 5.1: Standard Deviations Recorded at Two-weekly Intervals. These examples are for selected agents all classified as Type 7, taken from the beginning of a run. These patterns were found to continue in some cases for a long period.

days. Simulations run over longer periods of three to five years showed that the final difference in standard deviations is very little.

On investigation, it was found that the K_α values for the first four agents ranged from 0.9900 to 0.9998. The standard deviations for these, although generally declining, go through periods of temporary increase. In contrast, the two final entries in the table, representing the pattern of the majority, showed a steady decline. At 200 days, the figures for these were negligible, while those for the first group were still at a level which, although now manageable for practical purposes, are comparable to levels reached by the majority in the first few weeks.

These results suggest that these agents border on a separate type, or subtype, $1 = K_\alpha < K_\beta$. This group shows behaviour with many of the characteristics of Type 8 agents. There are a number of agents of Type 8 with similar behaviour, the two together forming a marginal strip where standard deviations are higher and more sensitive to movements in price than they are for most Type 7 agents but not to the same extent as the majority of Type 8.

Early problems were also encountered by some members of Type 4. These were very few in number but interesting to note. They approximate a further type described by Lad [42] as Type 9, with $K_\alpha = K_\beta < 1$, and border on Type 1. Similar to these, the mean of the precision may fall very

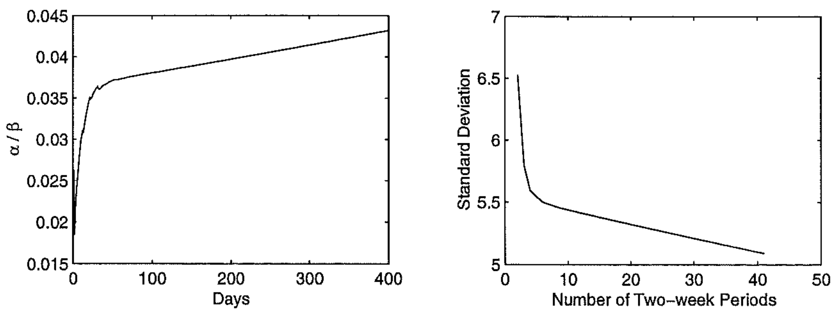

Figure 5.3: The Mean of the Precision Compared to Standard Deviations for an Agent of Type 9. This agent's values of K_α and K_β are 0.9117 and 0.9121, respectively.

low but, in these cases, does not converge to zero. Since K_β is greater than K_α even though the difference is slight, the mean approaches a level that remains apparently fixed over long periods although, in fact, it does rise very slightly. An example of the precision mean generated for one of these agents with its associated standard deviation curve is illustrated in Figure 5.3.

5.1.2 Variations in Sensitivity

In agents with both K_α and K_β greater than 1, a greater sensitivity to the magnitude of changes in price is also observed, especially when the market becomes more volatile. These changes, transferred through the expression $x_1^2/2$, incorporated in the updating of β, naturally affect the mean and the variance of the precision for all agents but when K_α is below 1 this has little perceptible effect. When $K_\alpha > 1$, however, such changes in price may come to dominate the agent's perceptions. Small variations in price will feed through to the agent's bid-ask spread to be expressed as small standard deviations while volatile prices may lead to wildly fluctuating standard deviations.

In one test run, interaction parameters were set high in order to induce a crash. While the majority of agents, although reflecting the general uncertainty, were able to maintain values of sigma well below 0.05 with many less than 0.001, the more sensitive agents of Type 8 registered standard deviations as high as 28.5. Such a degree of lack of precision incapacitated

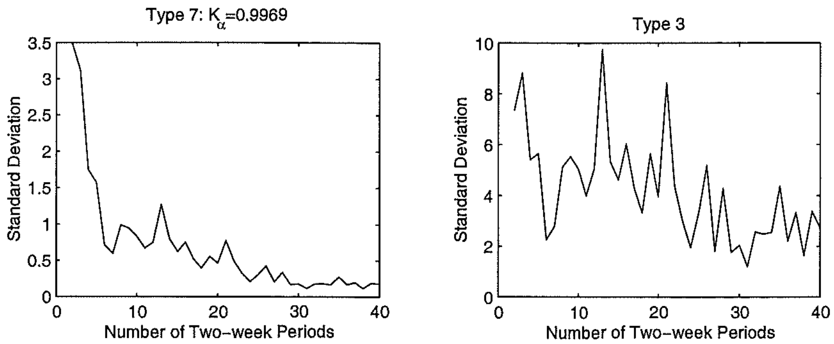

Figure 5.4: Recorded Standard Deviations Showing the Effects of Volatility in Individuals of Varying Susceptibility.

such agents completely during the crisis. Variance for Type 3 agents was even higher, a result only to be expected since these have a value of c_1 above unity. This of course was an extreme situation, but serves to highlight the vulnerability of agents with both precision parameters in excess of unity. By comparison, the reactions during this period of crisis for those forming a sub-type close to $K_\alpha = 1 < K_\beta$ ranged from 4 to 9.

Figure 5.4 shows the contrasts in the recorded standard deviations of two agents. The first example is an agent of Type 7 with a K_α value of 0.9969, the other is a Type 3 agent. The disadvantages, even for the borderline Type 7 agent, can be seen by comparison with the more typical example shown in Figure 5.2.

A progressive increase in sensitivity is observed from the transition region between Types 7 and 8 and continuing through to Type 3 as values of K_α and K_β draw closer together. We have already excluded those of Type 3 with $c_1 > 1.5$, so that those remaining, while not holding firm views on the unpredictability of prices, may nevertheless feel at times that they are on shaky ground. It is not surprising, therefore, that agents of this type show a more pronounced reaction to sudden movements in price.

It has been noted that, in any ordered listing of standard deviations, the higher values are practically monopolised by entries for Type 8 and Type 3. They are also poorly represented in proportion to their numbers in check lists of either market or limit orders. This is particularly notable for Type

8 of which there are a large number of representatives in the model, 22%.

As would be expected, these agents with K_α very close to 1, with behaviour similar to that of agents of Type 7, do not appear so frequently on the lists containing the inactive agents. These have very low values for c_1. Others with slightly higher values, in the vicinity of $c_1 = 0.3$ and upwards, may appear, but only for a limited time. At the other extreme, if $K_\alpha \simeq K_\beta$, c_1 is close to unity and the agent is prone to those uncertainties faced by agents with $K_\beta < K_\alpha$.

5.2 A Question of Doubt

5.2.1 A Decreasing Doubt Factor

For a number of agents the curve for α increases without limit, for others it converges to some real number. In the case of the former, the curve for the variance of the precision can take one of three forms:

1. It may increase without limit, as is the case for the precision types 5 and 7,
2. It may converge to zero, as in Type 4 and in the boundary type, 9,
3. It may converge to a fixed quantity, as in the transition type 6, where $K_\beta = \sqrt{K_\alpha}$.

For the first group, the question must arise whether the confidence of the agents in the precision of their forecasts may be transient. The doubt factor, Equation 3.25, simplifies to $1/\sqrt{\alpha}$, where α is, of course, regularly updated and modified by the value of K_α. Since, for the set of agents under consideration, K_α is less than 1 and α is always in excess of unity, it follows that the sequence, $1/\sqrt{\alpha}$, must ultimately decrease in all cases diminishing the doubt factor as time passes. It follows that, after initial periods of varying length, these agents become confident enough to attempt the regular placing of profit orders. If they can forecast well, there is no reason, as far as this factor is concerned, why this should not continue indefinitely.

Even for the sub-types where standard deviation difficulties are arising for more or less extended periods, the factors influencing the lowering of the

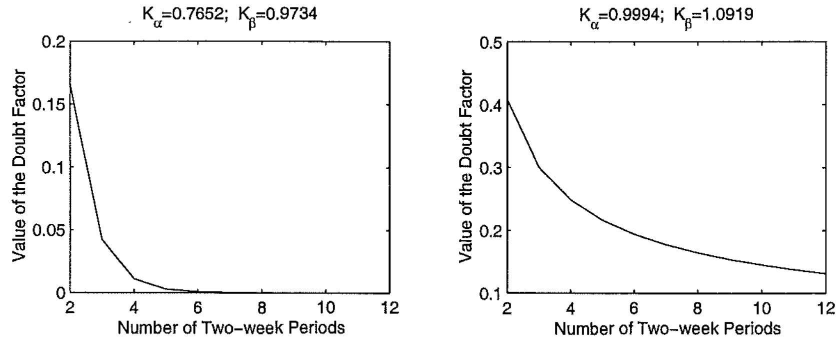

Figure 5.5: Variations in the Rate of Decrease of the Doubt Factor. Even the second example can place orders without hesitation by the eighth week of trading.

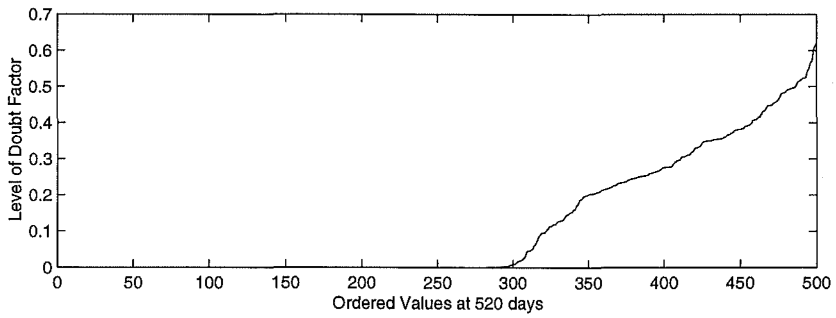

Figure 5.6: Long-run Values of the Doubt Factors of Agents of the Experimental Set. Agents are ordered from lowest to highest according to the levels of their doubt factors at a time (520 days) when these levels have settled into their long-term patterns.

mean of the precision are also reflected in its variance so that a nice degree of certainty is maintained for these agents.

Figure 5.5 shows the decreasing doubt factors for two agents, the first being of precision Type 5 which moves to zero very quickly. The second example is the agent belonging to the sub-type of Type 7 described in Section 5.1.1.

5.2.2 A Constant Doubt Factor

When the curve for α begins to change from convex to concave so that it will sooner or later converge to some value, its position relative to that of the

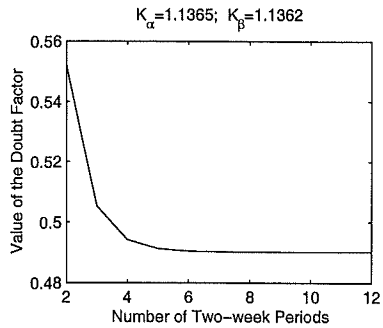

Figure 5.7: Record of the Doubt Factor for a Type 3 Agent with Precision Parameters Close to the 45° Line

similarly converging β becomes important for the variance of the precision and for the level at which the doubt function becomes constant. Figure 5.6 gives the long-run picture of the ordered doubt values for all 500 agents of the experimental set. Approximately 300 agents are seen to be free from uncertainty as measured by this criterion. The sharp rise in values corresponds with the change in the α-curve. At first we have the transition values occurring where K_α is approaching unity from either direction and where the doubt is still comparatively low, ranging approximately from 0.07 to 0.15. We recall that orders are placed on the limit order lists automatically for a doubt factor of less than 0.25, and avoided where it exceeds 0.5, and that a random process simulates the hesitation and discussion that can be assumed for values between these figures. These levels were set after experimentation, so that the only agents affected in the long run are those falling in the Upper Quartile of doubt values. A mere 3% of the population are excluded by their doubts from ever placing a limit order. Figure 5.7 shows the doubt factor for an agent of Type 3. It can be seen that this agent will never place an order without hesitation but, on the credit side, after a short introductory period will never reject such an order outright.

Since the refinement of the population, traders belonging to Type 3 comprise only 7.2% of the population. Although these have the highest doubt factors so that those too uncertain to place profit-taking orders at all belong to this group, close to 60% of them do consider it and some even do so without hesitation. In a number of the experiments made, both the levels of trading and of wealth for many Type 3 agents were found to be close to those of Type 8, justifying the retention of these agents in the population up to this point.

5.3 The Agent as a Profit Maker

5.3.1 Some Preliminary Tests

In assessing the relation between the attributes of an agent and the wealth acquired by him, it is necessary to study the combined effects of the location and precision parameters. While the prediction of tomorrow's price is strongly associated with the agent's outlook as represented by his K_μ-value, it has been noted that the magnitude of K_μ has been more highly correlated with success in prediction than with the accumulation of wealth.

In order to make further tests of the relationship between the accumulation of wealth and K_μ, agents in the experimental set were first categorised according to the magnitude of their K_μ values into the two groups, $K_\mu > 1$ and $K_\mu \leq 1$. ANOVA tests of differences in the mean levels of wealth between these two groups, with differences in precision ignored, showed a significant difference in the means ranging from the 2.5% to the 1% level.

This was not considered a fully adequate partition, however. When individual wealth levels were examined, it was found that much of the successful trading was being done by agents with values closer to unity, especially those less than 1. This central group contains those considered as Bayesian learners, a group described extensively in the models of the Efficient Markets approach. These are the rational learners of many models in the literature and many of these have had undoubted success in the current simulations, an example being the fourth agent whose forecast is reproduced in Figure 4.5. While an agent such as this one may miss many of the short, sharp movements in price and therefore does not score so highly in prediction, frequent trading with more conservative levels for limit orders are often seen to be producing better results than those whose predictions fluctuate more widely. Tests of this group were made possible by further partitioning into three groups containing high, low and medium values of K_μ. These tests gave the most significant results in comparisons between the extreme values, confirming the observations made in Chapter 4. When the medium values were involved, more distinction was found between high values and values closer to unity than between these and the low values.

Further ANOVA tests were then used to compare the precision types irrespective of the values of K_μ. These showed a significant difference between the means of the different types for both wealth and for activity, as measured by the number of transactions. Especially high values of the F-statistic occurred for the latter.

Records for both these variables were accumulated over a number of runs and compared. With a high degree of consistency, Types 5 and 7 showed the highest mean level of wealth. When others sometimes held this position, the reason was usually traceable to the tendency on the part of individuals of these two types, especially of Type 7, to experience large losses on a few occasions. On the other hand, Types 5 and 7 most frequently reached the maximum values. Type 3 agents are notable for a slow rate of accumulation of wealth, although losses are also of lesser magnitude. Types 5 and 7 also consistently recorded the highest number of transactions, followed closely by Type 4, frequently five or six times those of either Type 8 or Type 3. This clear-cut partition obviously bears a close association with differences in the levels of standard deviation and of the doubt factor for these types.

5.3.2 Refinement of the Types

The number of types was now extended by subdividing each precision type according to whether the accompanying K_μ was above or below 1. The types that resulted were labelled with an extra digit, '0' for $K_\mu \leq 1$ and '1' for $K_\mu > 1$. Type 7, for example, was now studied as Types '70' and '71'.

Significant differences between the groups were again found, the values of the F-statistic being generally higher than before – frequently well beyond the 0.1% level of significance. Also, the wide spread of values apparent in types with low standard deviations and doubt factors was now partly explained. Where these latter properties were not inhibiting trading activity, those agents with $K_\mu \leq 1$ were regularly recording the minimum levels of wealth of each pair of precision types as well as a higher number of transactions in each. Mean values proved to be higher for all types with $K_\mu > 1$. Maximums were variable, but those with both precision parameters below 1 appeared to be able to achieve a better result with lower values of K_μ. Agents with $K_\mu \leq 1$ also traded generally twice as frequently within each group as those with $K_\mu > 1$.

Type 4 now showed more potential, high maximum levels being common among those of Type 41 while the Type 40 agents showed low minimums. The mean wealth of the former was, therefore, frequently found to be at a competitive level. Type 50 shows heavier losses than Type 51 but did not fall as low as Type 70. However, due to the latter's larger market share, this fact may not be of great importance in the evolutionary process. Type 71 typically showed a wide spread of values, frequently recording both the highest maximum and the lowest minimum in the same run.

The above results have been taken from short simulated runs, between one and two years in length. At this point in time, the Type 81 group often gave the highest mean level of wealth of any type in this partition, although very seldom appearing as maximum or minimum. This was due to losses sustained by other groups in this early trading, so that their comparative inactivity proved to be a temporary advantage for this group. Many of Types 3 and 8, of course, trade very little, if at all, until they have survived a period of adjustment. Type 31 agents were found to be steadily increasing their levels of wealth but at a much slower rate than other types.

To sum up, while the bulk of trading was being done by agents with $K_\alpha < K_\beta < 1$ and $K_\mu \leq 1$, these were frequently suffering large losses so that the highest mean values of wealth were often found where K_μ was greater than 1. The one exception to this was Type 50, which often attained the highest mean in spite of a higher number of losses than any type but Type 70 and fewer high levels of profit.

A third subdivision of the precision groups was now made to define a separate group between $K_\mu = 0.95$ and $K_\mu = 1.05$. These limits were chosen both for the levels of K_μ and to maintain adequate group sizes for valid comparisons to be made. The medium group was denoted by the digit

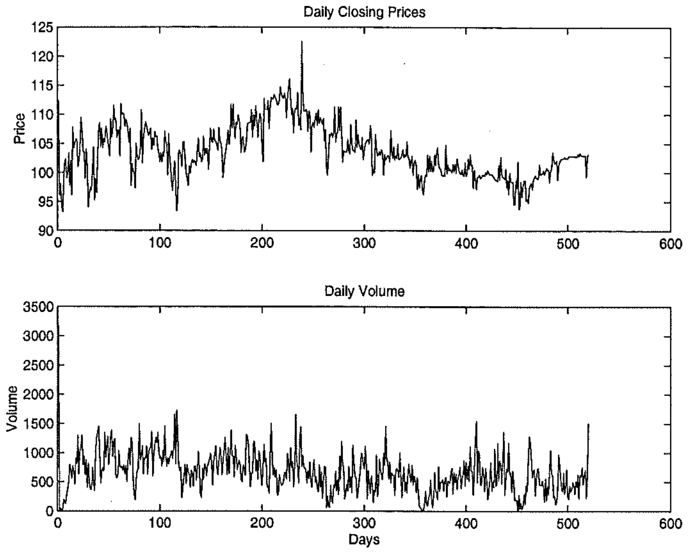

Figure 5.8: Daily Price Series and Volume of Trade

Type	Mean	Median	Minimum	Maximum
40	$19942	$19815	$18890	$21030
41	20824	20592	19369	22329
42	20237	20310	18188	21607
50	20018	20030	17518	21676
51	20510	20426	19134	22704
52	20777	20510	18922	23127
70	19835	19887	16938	22481
71	20522	20538	16611	22862
72	20618	20637	18228	22920
80	20408	20322	18717	22279
81	20695	20608	20108	21692
82	20373	20358	19165	21539

Table 5.2: Comparative Statistics for a Partition of Agents into Types. Figures show the value of portfolios combining both cash and shareholdings at the conclusion of the run. Remember that every agent began with a portfolio of $20,000.

Type	Mean Number of Trades	Standard Deviation
40	26.368	25.8
41	11.857	7.66
42	10.412	7.34
50	21.158	17.5
51	15.267	4
52	16.76	5.75
70	28.306	20.35
71	15.164	6.92
72	17.21	8.8
80	12.088	15.12
81	4.14	3.14
82	5.75	6.98

Table 5.3: Comparison of Distributions of Trading Activity for Grouped Agents. Activity is measured by the number of transactions concluded.

	260 Days	520 Days
Comparisons	F-Value	F-Value
'0'–'2'	4.464	4.441
'2'–'1'	0.7941	1.559
'0'–'1'	7.342	8.377

Table 5.4: ANOVA Test Results for a Typical Run to Compare Eight Groups.

$F_{7,200}(0.95) = 2.06$	$F_{7,1000}(0.95) = 2.02$
$F_{7,200}(0.99) = 2.73$	$F_{7,1000}(0.99) = 2.66$
$F_{7,200}(0.999) = 3.65$	$F_{7,1000}(0.999) = 3.51$

Table 5.5: Values of the F-distribution. Values in Table 5.4 are from an $F_{7,499}$ distribution.

'2' appended to the original type number, for example, '70', '72', and '71'. Type 3 was omitted from these tests due to the small sizes of sub-types that would have resulted.

In the new results now generated, groups coded '0' were still holding the lowest minimum levels of wealth, with Types 51, 52, 71 and 72 achieving the highest maximums. As well as showing low minimum values, it was not uncommon to find that close to 50% of agents of Types 50 and 70 had failed to show any profit at the end of the period tested.

An Example

Table 5.2 summarises the financial position for one run of this subdivision and Table 5.3 the number of transactions for each type and the spread that has occurred within the types. The run was for two years, with shocks set to occur at intervals of 25 days and interaction parameters set at $\lambda_1 = 0.3$ and $\lambda_2 = 0.35$. The resulting series of daily closing prices, illustrated in Figure 5.8 together with a record of the volume of trading over the same period, shows an alternation of minor peaks and troughs providing ample opportunity for the making of profits. The volatility of the early months illustrates the natural uncertainties of the settling-in period. The final part of the series shows the effects of the greater degree of precision in expectations that develops over a population where there is no movement of agents in or out of the market.

The extent of trading remained as before with the largest number of transactions occurring within each precision type being for those with low K_μ and the lowest for high K_μ with the intermediate group placed midway between the two. The same ranking also applied to spreads within the number of trades. Table 5.4 gives the results of ANOVA tests for this run of the simulation at both one year and at two years. Some relevant values of the F-distribution for comparison are given in Figure 5.5.

5.3.3 Conclusion

The experiments made with different combinations of parameters have given the following indications:

1. that differences between the types exist, but

2. that conflicting scenarios can be generated by the unique interactions that take place during any particular run, together with influences both internal and external. For example, an advantage may accrue to an agent by his initial level of expectation or the fortuitous occurrence of a piece of unexpected information.

It has become clear that, for any long-term assessment of success, there is a need to set up a process that will allow the population to speak for itself by displaying its evolving patterns. This process must be based on a set of criteria that will represent the decisions of agents to enter or leave the market and that will determine when these decisions should be put into effect. These criteria will need to take into account the variety of patterns possible in the financial histories of those engaged in speculative trading so that all agents are enabled to make the most of their opportunities.

Chapter 6

Simulating an Evolutionary Process

6.1 The Criteria of Success

6.1.1 The Problems

It has been shown that lack of success in a speculative market is reflected in one or more of the following forms of behaviour:

1. Persistent inaccuracy in forecasting,
2. Prolonged non-participation that arises from an agent's uncertainties rather than from his maximising strategy.
3. Failure to maintain positive profits at an acceptable level.

It is assumed that an agent would monitor his forecasting record only when seeking the cause of financial losses, and, when unable to improve his predictions, would then withdraw from the market. There is, therefore, no need to test for forecasting weakness as a separate criterion for leaving the model. Regular testing of the agents' financial positions should be sufficient.

One of the major questions to be addressed in the simulation of an evolutionary process is that of the inactivity of some agents who, due to the excessive width of their bid-ask spreads, find themselves at a disadvantage in competing with other agents. In some cases, this unplanned inactivity extends over long periods but, in others, it is found only during periods of varying length after agents have entered the market for the first time.

There are two ways of approaching this problem in a simulation. Because in many cases the difficulty is not a permanent one, we may decide that our criteria must not be so rigid as to exclude possibly successful agents who may need a longer period of introduction to the conditions of the market. For example, it can be seen that a premature move to exclude a considerable subset of agents, such as those described in Section 5.1.1, on the basis of their temporary inactivity may be a mistake. It may be better to defer such a move until a full year has passed. If these agents are not destined to succeed they will be forced out in time in any case.

It was further noted that sharp but temporary losses were frequently a feature of agents' financial histories. With no withdrawal of agents in previous trials of the simulation, such losses were often remedied within a reasonable period of time.

Understanding that forecasting inaccuracies may sometimes be counteracted by a change in the direction or rate of movement of a trend, and that for many types both bid-ask spreads and doubts decrease over time, it is necessary to determine some standard for the tolerance of temporary losses. With a declared goal of short-term profit-making, to what extent can the effects of learning leading to a greater degree of certainty be allowed for?

A similar question applies to the level of profits that is desired. At what rate are profits expected to be made when the activities simulated are to be extended long enough to isolate the characteristics that are likely to evolve in a set of traders over an extended period of time?

Several experimental approaches are indicated:

1. **The Timing of Replacements**

    (a) An approach that allows enough time for agents who experience an initial period of uncertainty or who have suffered losses through an unfortunate transaction to remedy this situation.

    (b) A system based on regular and frequent accountability in which even short-term losses or delays are unacceptable.

2. **The Level of Profit-making**

   (a) An approach in which the measure of success is no more demanding than a positive degree of profit.

   (b) A more realistic approach in which the decision to continue trading is guided by the need for a continuing accumulation of profits.

A further simulation problem concerns the method of procedure to be adopted in the replacement of agents in the model. In a real world market, agents move in and out of the market as dictated by their individual histories and influenced by their individual attitudes towards the risk of further losses as balanced against the chances of future success. This movement of agents is a continuous process. For the purposes of this simulation, it is convenient to test the progess of all agents at specified intervals in order to select those who do not, at that point of time, meet the predetermined criteria. In this way, distributions of the characteristics of successful agents can be formed and agents seeking to enter the market can be categorised accordingly. Systematic records can be kept, comparisons made and statistics collected to illustrate the progressive development of the market over time.

6.1.2 The Solution – Three Versions

The total value of each agent's wealth has been routinely evaluated in the course of daily updating. To implement this part of the simulation, these totals are copied at six-monthly intervals to another column of MK3 and retained there so that comparisons can be made after a further six months trading has been completed.

In the implementation of the most tolerant version, which will be referred to as *Version0*, each agent's profit is tested at six-monthly intervals from 260 days onwards. Where total wealth currently shows no increase on the initial value of $20,000, the last period's total is also examined. If this also is below the limit, the agent prepares to leave the market. In order to accommodate those agents who may be experiencing a longer initial period of adjustment, the first two tests, at 260 days and 390 days, require only that the level of wealth does not fall below $20,000. In subsequent tests a specific level of

No. of Days of Participation	Critical Value
130	20000
260	21000
390	21000
520	22050
650	22050
780	23152
...	...
...	...

Table 6.1: Section of TableW. This extract covers the period from 6 months to 3 years.

positive gain is required. By referring to the recorded total of days that an agent has been part of the system, it is ensured that the same conditions apply to all new entrants at whatever stage they enter the model.

The next version, *Version1*, retains the same requirements as Version 0 in that two consecutive failures form the criterion for leaving the market. However, while Version0 needed no more than an ability to avoid losses taking the total below the original level of $20,000, Version 1 is based on a progressive annual accumulation of profit of 5%. This amount is reasonable to require in that the stock does not pay dividends. To provide the necessary comparisons for a population of agents who have been functioning in the system for varying periods of time, a reference table, *TableW*, was stored listing the six-monthly levels of wealth required for such an increase. A portion of this table is shown in Table 6.1. Column 1 of this table lists the number of days of participation in the market in six-monthly intervals. At each assessment, the number of days is located in this column for each agent in turn. The corresponding figure in Column 2 then indicates the limit that applies to him at this stage. The critical limit for the six-monthly lagged value is then found in the previous entry of the table. Since the increment is calculated annually, agents who are new and inexperienced are not penalised here.

Version2 uses the same profit criterion as Version1, with tests again taking place at six-monthly intervals starting now from 130 days. The critical difference is that no allowance is made for any short-term fluctuations that may take total wealth below the level required by a 5% annual increase. It may be noted that, in practice, many agents do exceed this rate of increase. However, this has not been incorporated in any testing in this thesis.

Application of these methods results in a list of those agents who will leave the market. These agents' entries in the matrix, MK3, are then marked for replacement and procedures are set in motion for them to leave the market.

6.2 The Evolutionary Process

6.2.1 An Overview

The process of evolution will first be summarised in brief. The simulation is run for the predetermined interval. At the appointed stopping points, the following operations are performed by the experimenter.

1. The current wealth level of each agent in the experimental set is tested to distinguish successful from unsuccessful participants according to the version chosen.

2. A set of replacement agents is chosen, the aim being to imitate the characteristics that predominate among those displaying the best performance in recent trading.

3. Unsuccessful agents are replaced and appropriate adjustments made to share ownership and to lists.

4. Information is recorded for use in the analysis of the experimental set as it develops over time.

The simulation of daily activities then continues through the next interval. However, the background population of additional agents must also be renewed at times in order to provide a realistic environment in which the experimental set may operate. We are not concerned with the success or failure of these agents or their reasons for leaving or entering the market. It is sufficient that regular changes are made in this population without interrupting the smooth flow of trading. Replacement of small groups of these agents are planned to occur at times which do not coincide with changes in the experimental set.

6.2.2 The Analysis of Agents

Once profits have been assessed according to the version in use, agents in the experimental set who have been successful are distinguished from those who are prepared to leave the market at this point. Records for these two sets of agents are now analysed separately. An explanatory flowchart is given in Figure 6.1.

The Successful Agents

1. **Step 1** The current distributions of values for each of the parameters in the set, $(K_\mu, K_\alpha, K_\beta)$, are first examined, the aim being to imitate, in the distributions chosen for incoming agents, the characteristics that predominate among those displaying the best performance in recent trading. From the recorded frequencies for each parameter for the successful agents, distributions are formed. Changes in these distributions over time are noted, with particular reference to any variations in their location or spread.

2. **Step 2** Continuing the analysis of these agents, a table is set up showing the proportions belonging to each type as currently defined. Successive updatings of this table are used to give a picture of the increase or decrease of each of these types in the long-term evolution of the market.

Agents Leaving the Market

Agents designated to leave the market at this point are now processed so that two necessary actions can be taken:

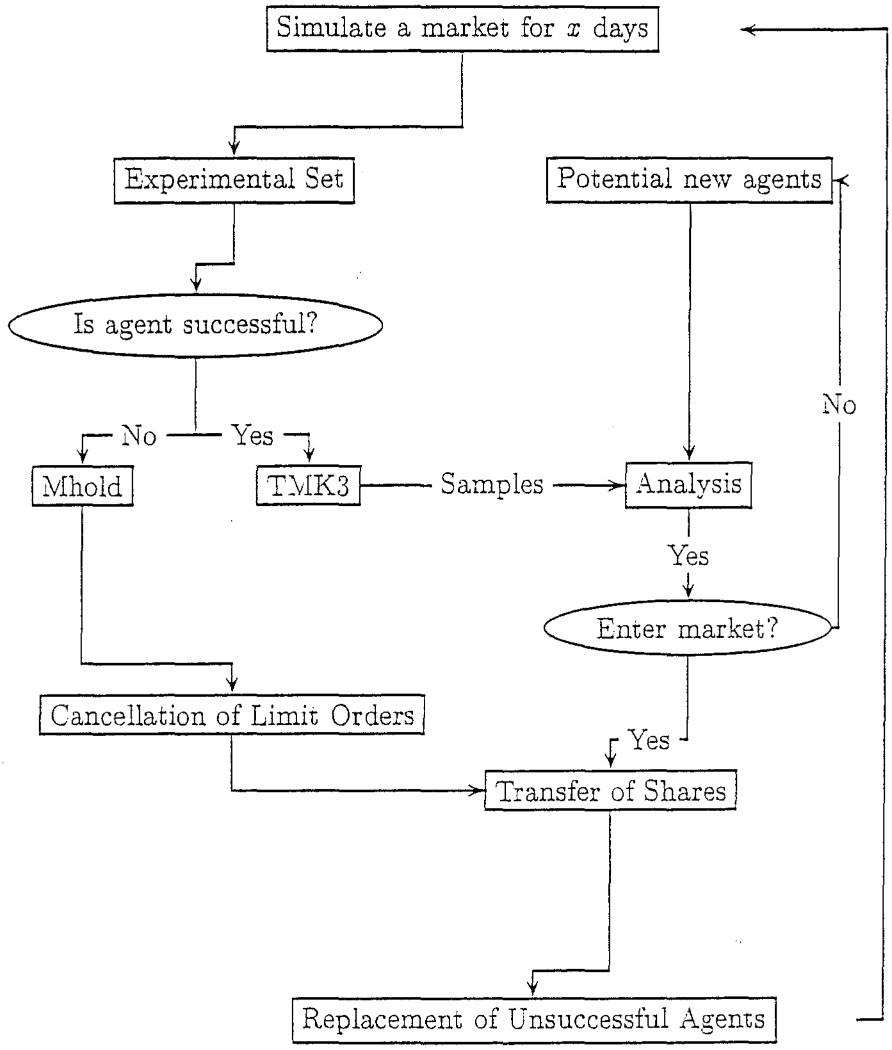

Figure 6.1: The Process of Replacement of Agents. The units *Mhold* and *TMK3* represent a partition of MK3 into unsuccessful and successful agents respectively.

1. **Step 1** To prevent unwanted variation in the total share issue, the total number of shares still held by agents who are withdrawing must now be assessed. These will be distributed equally among all the new entrants at the latest closing price with cash balances adjusted accordingly.

2. **Step 2** Any outstanding limit and stop-loss orders held by these agents are noted and removed from the lists.

6.2.3 The Replacement of Agents

1. **Step 1** A temporary holding matrix, K3, is set up for the storage of potential entrants to the market. This matrix is of the same structure as the original MK3 with the same number of columns and a number of rows large enough to provide the number of replacements needed after tests for type have been made and the relevant proportions of each type represented in the composition of the set of successful agents allowed for.

2. **Step 2** A procedure now builds the matrix, K3, into a source of agents prepared to enter the market. Random draws are made for the formation of a set of parameters for each potential incoming agent. These parameters will then conform to the patterns determined by the successful agents. The type of each of these new sets of parameters is then recorded to conclude their definition. Values in the new matrix are now initialised. Total starting capital for each agent is \$20,000 and starting values for μ_0 and its variance and the parameters K_μ, K_α and K_β follow the pattern of the original initialisation. The new agents entering the market do not show the same effects of experience as those who have already been trading for some time. Standard deviations, for example, will display those initial uncertainties that have been commented on in Chapter 5. Thus, at each stage, a population is formed that, as well as having intrinsically varied learning approaches, are also at different temporal stages of this learning process. This is a realistic situation which provides an environment in which adaptation is needed and in which opportunities abound.

3. **Step 3** Agents leaving the market are now replaced in MK3 by selection from K3. The identification number of the agent being replaced is transferred to the new entrant, and the cells counting transactions and daily participation are reset to zero. Those shares formerly owned by retiring agents are issued to each new agent at the most recent closing price with the agent's cash holding adjusted accordingly. MK3 is now ready to continue with the simulation.

6.2.4 Conclusion

All 1000 agents participate equally in the activities of the market, but detailed records are restricted to the test group only. These records include:

1. Progressive levels of K_μ, K_α and K_β, which are recorded after each replacement of agents in the experimental set. Measurements are made of mean, median, variance and of maximum and minimum values for the distribution of each parameter.

2. The proportions of each type of agent present in the market at each six-monthly interval,

3. The proportions of the total wealth of the experimental set held by each type defined.

4. Periodic listings of the comparative levels of wealth obtained including the means and maximums and minimums for each type and the highest and lowest figures for individuals listed together with their type category and K_μ-value.

5. The final values of portfolios for all agents for comparison of the success of the experimental set relative to the background population.

6. The series of prices and of the volume of trade generated by the simulation so that the configurations can be studied in combination with the changes that may have occurred in the experimental set.

7. Other details of use in the assessment of the results such as the variability in the number of agents leaving the market at each stage and

the ratio of high to low values of K_μ associated with the levels of wealth in the Upper Quartile.

Chapter 7

The Dynamic Evolution of the Market

7.1 Introduction

This chapter will record the results of a series of experiments in which the criteria of the last chapter were applied, modifying the structure of the Experimental Set by the withdrawal and replacement of unsuccessful agents. The aim is to discover, by a continuous process of renewal, the composition, in terms of the parameter set $(K_\mu, K_\alpha, K_\beta)$, of those traders who profit most from speculative trading. Agents are to be studied now as a group rather than as individuals.

Since the main topic of study in this work has been the effects on behaviour of the set of parameters, $(K_\mu, K_\alpha, K_\beta)$, the values of these have been kept constant throughout for each individual agent. The values describe the learning process engaged in by each individual and it is assumed that this is not revised by him over time. A different study without this assumption would be interesting but is not attempted here. The market as an entity, however, is seen to learn, with the distribution of these parameters across participating agents changing over time. Those who are failing to adapt leave the market, while those entering are encouraged to do so because they have been observing those who are proving successful and believe that they too possess these same winning qualities. As time passes, the group therefore takes on a characteristic form, emphasising certain combinations of attributes rather than others. The proportions of each of the population types described in Section 5.3.2 change over time. These proportions will be referred to as *market shares.* In many runs of the simulation these market shares have been shown to correspond closely to the actual holdings of the types concerned, but this is not always the case. Percentages of market shares quoted and the distributions of $(K_\mu, K_\alpha, K_\beta)$ shown are those applying to the Experimental Set only. The results of these experiments are now presented as they relate to several main questions:

- What changes, if any, take place over time in the distributions of the parameters K_μ, K_α and K_β ?
- We have seen in Chapter 4 how different combinations of these parameters form distinct types. Do the proportions of the population belonging to these various types alter?
- Does the experimental set show any tendency to increase their profits when compared with the background population?
- Do short-term requirements necessarily lead to a healthy market in the long term?

7.2 Results for Version 0

7.2.1 Distributions and Market Shares

This version, requiring only that agents do not suffer losses below the monetary value of their portfolios at the time of entry to the market, resulted, on the whole, in minimal changes in the population of the Experimental Set. The constraints are undemanding and market shares of the different types were frequently stable over long periods.

There was some variation in the results over the ten runs that were executed, and some similarities across them. Six different settings of $(n1, \lambda_1, \lambda_2)$ were used in the testing of each version: (500, 0.01, 0.01); (50, 0.05, 0.1); (25, 0.3, 0.35); (25, 0.45, 0.35); (15, 0.5, 0.35); (4, 0.55, 0.65). Occasionally,

Type	Percentage at 0 Years	Percentage at 10 Years		
		Ex.2	Ex.1	Ex.3
3	7.2	1.8	6.4	0.8
4	10	11.4	9.2	18.8
5	14.8	24.8	19.2	44.8
7	46	54.4	46.6	32.8
8	22	7.6	18.6	2.8

Table 7.1: Percentage of Market Shares for Selected Runs of Version 0 for the precision types. (See Figure 4.9.) The reason for the ordering of Examples 2,1 and 3 will become apparent in the text. Example 1 is a model type run yielding the most common and intermediate type of result.

to study the crisis situation of a crash, (4, 0.75, 0.85) was used. In all runs, market shares of Types 3 and 8 decreased and that of Type 50 increased. Results ranged from one in which there was a substantial increase for Type 7 to several with a marked increase for Type 5. Runs varied in length but, for purposes of comparison, Table 7.1 was compiled from levels of market shares taken at the completion of 10 years for three different runs. Type and initial proportions are also shown. The majority of tests gave results similar to those in Column 4 (headed Ex.1) of the table with Types 3 and 8 showing a small decrease, Type 7 a minimal change and a small increase for Type 5. Type 4 usually varied within a small margin of its initial value.

Example 1

Results over a longer time span are illustrated in a continuation of the run from which the figures in Column 4 are taken. This run was for 22 years with interaction parameters set at $\lambda_1 = 0.3$ and $\lambda_2 = 0.35$ and shocks timed to occur at intervals of 25 days. The price series generated by this run is shown in Figure 7.1.

The number of agents in the market with both precision parameters above 1 decreased markedly. Those of Type 3 completed the run with only 2.6%, while Type 8 fell to 9.2%. For both types, the initial proportions were

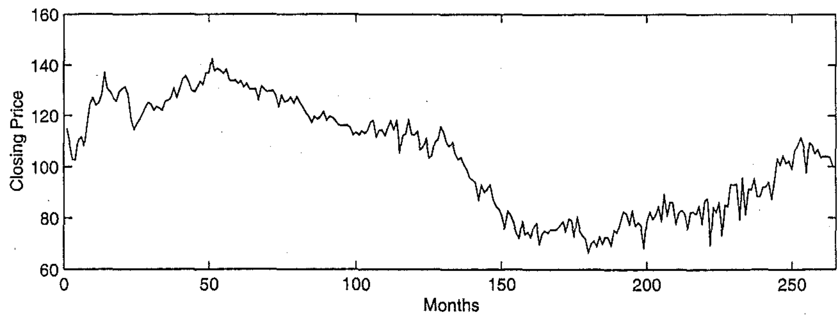

Figure 7.1: Monthly Price Series for Example 1

maintained until the decline in prices occurring around 10 years, when they fell quickly to the above figures with very little subsequent change. This was despite the fact that prices are seen to rise back to the initial price level over the next twelve years. Other runs produced very similar results for these types. The value of K_μ appeared to produce no consistent variations here. It was interesting to note that 70% of the Type 8 agents who survived for the complete run of 22 years had values of K_α close to 1, thus belonging to the borderline sub-type discussed in Section 5.1.1.

Where both precision parameters were below 1, an overall increase in market share occurred. This was most marked for Type 5, which rose to 28.6%, virtually doubling in size, with most of the change occurring where K_μ was below 1.05, that is, in Type 50 and, to a lesser extent, in Type 52. Type 50 underwent little change while prices remained high but increased rapidly during the trough thus achieving a higher level which was then retained until the end of the simulated run. There were no runs using Version 0 in which Type 50 did not increase its share of the market. Frequently, there was an initial increase in the case of this type which, once established, did not diminish. The extent of these changes was limited, however. No type disappeared from the market and the proportions of others remained stable over extended periods. In this example, for instance, the 22-year levels were already established by 15 years. Type 7 completed the run holding the same percentage of the population as it did initially, 46%. Type 4 increased to 13.6%, with Type 40 experiencing the largest changes among those of Type

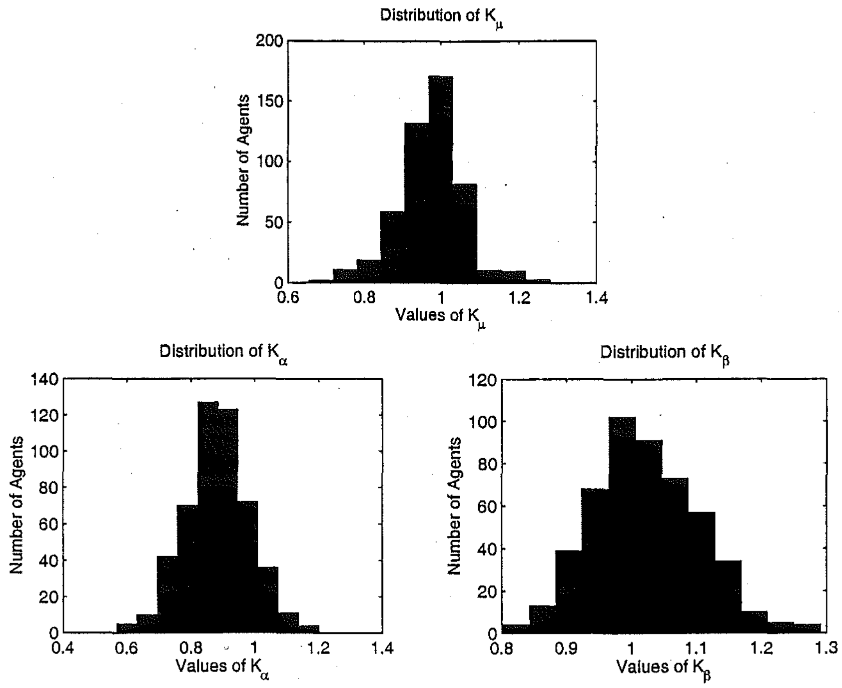

Figure 7.2: Final Distributions of $(K_\mu, K_\alpha, K_\beta)$ for Example 1

4.

The distributions of the parameters have undergone some changes as a result of this movement. The initial mean values of $(K_\mu, K_\alpha, K_\beta)$ for this and the majority of runs were given by (1.01, 0.95, 1.06). After the simulation had been run for 22 years, values had fallen to $(0.97, 0.88, 1.02)$. These new distributions are illustrated in Figure 7.2. The initial distributions were displayed in Figure 4.20.

The majority of runs for Version 0 showed movements very similar to those in this example. There was little change in the market share of the largest group, Type 7, but there was a fall in the number of agents with both K_α and K_β above 1 with a corresponding increase of those whose precision parameters were both below 1.

It has become apparent during the tests that a movement has taken place favouring agents with $K_\alpha < K_\beta < 1$, and $K_\mu \leq 1.05$ – a movement from right to left in the diagram of Figure 4.9.

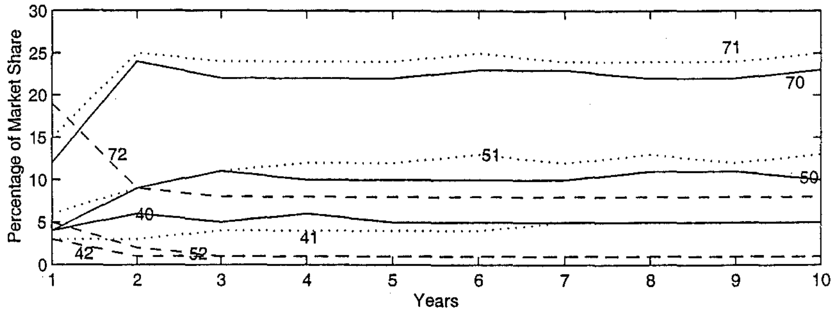

Figure 7.3: Patterns of Growth in Market Shares of Selected Types for Example 2

While the majority of runs gave results similar to the above example, there were some interesting examples that were atypical. Two of these are exemplified below in Examples 2 and 3. Similarities were that Type 5 always showed a strengthening hold on the market to a greater or lesser degree; that the proportions of Types 3 and 8 always fell but these agents never wholly disappeared from the market and that agents with lower values of K_μ were shown to have a degree of advantage.

Example 2

At one extreme was the run depicted in Column 3 of Table 7.1, headed Ex.2. This run was continued for a further six years giving a minimum of change in the situation shown by the 10-year figures. Type 7 continued to rise to 56% and Type 5 to 25%. This was the only run with such a high level of market share for Type 7 agents. The market share of Type 4 continued almost constant and those of Types 3 and 8 fell to 1.6% and 7%, respectively. This run was made with parameters $(n1, \lambda_1, \lambda_2)$ set to (25, 0.3, 0.25), and resulted in a series that was generally rising throughout with the final evaluation of portfolios made at \$130. The means of the final K-values were (1, 0.89, 1.03).

Although the results of this run differed from those of Example 1 in the levels of market shares reached, the patterns of increase were similar. As with the majority of runs using this less regulated version, once the relative levels of the types were established they were maintained with very little change over extended periods with few agents leaving or entering the market. This is illustrated in Figure 7.3, which traces the changes taking place over time for different types of agents. Levels are shown in percentages of the experimental set belonging to the main types studied at yearly intervals. The first 10 years only are included since there is no change after this point.

Although, in this example, agents belonging to Types 70 and 71 form a larger percentage of the market, the pattern seen here is characteristic of Version 1, occurring in all but a few extreme cases of which the next example is one.

Example 3

Several runs of this version resulted in higher percentages for Type 5, one of which is illustrated in Column 5 of Table 7.1, headed Ex.3. This was a shorter run with $(n1, \lambda_1, \lambda_2)$ set to (25, 0.3, 0.35), and ended at twelve and a half years with a sharp fall in prices to \$28.56 preceded by a period of marked volatility. This was not initially intended as a longer run and runs cannot be repeated exactly due to the random factors, but experience from runs in Versions 1 and 2 suggests that the market would have fallen into disarray soon after this point.

This run recorded the minimum market share level obtained in this version for Types 3 and 8, being 0.8% and 1.4%, respectively. Over 70% of the population were found to belong to those groups with $K_\alpha < K_\beta < 1$, and Type 7 fell to 26%. Only a small part of this percentage were Type 71 agents. Type 70 peaked as early as 6 years before losing ground, gradually at first but more rapidly after 10 years as the market became more volatile.

Mean levels of $(K_\mu, K_\alpha, K_\beta)$ for this run fell to (0.91, 0.75, 0.95).

This was not a result typical of the tolerant version, but it did occur once. The result shows a pattern that was to occur regularly and in a more extreme form when more constraints were placed on the measurement of profit.

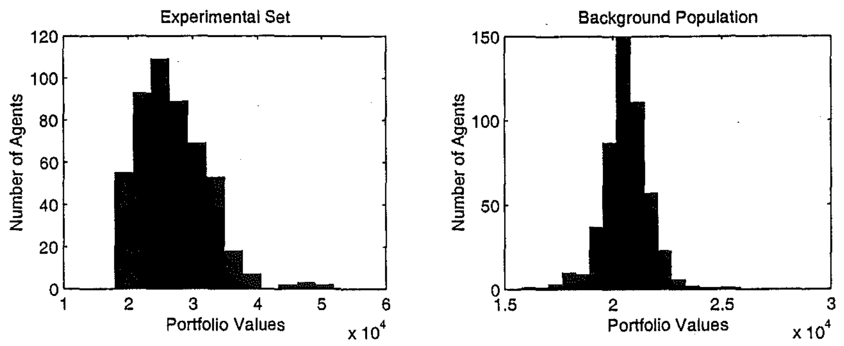

Figure 7.4: Distribution of Wealth for Example 1. Both the Experimental Set and the Background Population are shown. These distributions were recorded at 22 years at the latest closing price of $113.1.

7.2.2 A Survey of Profits

The question now remains: how has the altered composition of the experimental set contributed to an increase in profit-making? A comparison of the accumulated wealth for the experimental set with that of the background population for Example 1 is given in Figure 7.4. Means for the two groups were $27,007 and $20,585, showing a superiority of just over 30% for the experimental set. It is noted that the final price at which the stock was evaluated was $113.1. Also, in this run, there had been few changes in the composition of the experimental set since the 15-year mark, with very few new traders entering the market. This fact was favourable for the accumulation of higher values of wealth. Only 2.6% of the agents had portfolios valued below $20,000 at the final check of the run. It was interesting to find that 60% of the higher levels of wealth (above $40,000) were achieved by agents with values of K_μ within 0.003 of unity, the majority of whom belonged to Type 72.

The experimental set in Example 2, evaluated at $130.1, showed a 15% improvement over the background population with means of $25,464 and $22,106, respectively. The distributions of wealth are shown in Figure 7.5. Here there were more lower values and Type 70 acquired the highest proportion of wealth.

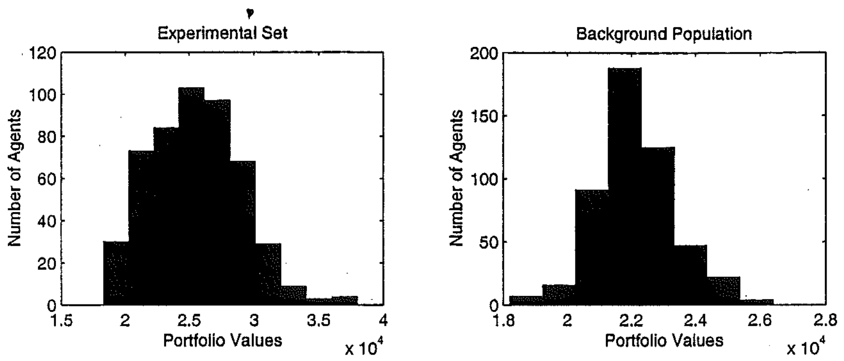

Figure 7.5: Distribution of Wealth for Example 2. These values were recorded at 16 years at a closing price of \$130.

7.2.3 Summary

The most likely outcome of a simulation using this tolerant version can be summed up as:

1. A slight relocation of the distributions of the K-parameters towards the left, with virtually no change in the spreads.

2. A trading population containing representatives of all those types initially choosing to enter the market but with some changes in the proportions of those with precision parameters that are either both above or both below unity.

3. Improved levels of wealth. The highest margin was found where less change occurred in the proportions of agent types in the market. The large increase in Type 5 occurred in only one instance, Example 3.

7.3 Version 1

Fifteen runs were made in the study of this version, with the same range of values for $(n1, \lambda_1, \lambda_2)$ as before.

7.3.1 Distributions and Market Shares

The more demanding requirement of Version 1, namely, that of a 5% annual increase in the total value of the agent's portfolio, was accompanied by a sharper discrimination between the types with consequent changes in the distributions of $(K_\mu, K_\alpha, K_\beta)$ over the experimental set. In this scenario, much more change took place. An increase still occurred in the percentage of agents with $K_\alpha < K_\beta < 1$, together with a lowering of K_μ, but the rate of these changes increased. Again, there was a range of variations between the runs (15 were made in all) but, in the majority of experiments, whatever the interaction parameters, Type 50 came to dominate the market with Type 40 as a kind of junior partner.

A noticeable feature of the simulation results run according to Version 1 was the disappearance at an early stage of Types 3 and 8. By two years, Type 3 was frequently no longer represented in the market. Type 30 sometimes persisted for three or four years but rarely beyond this. Unusual circumstances, such as the continuous boom conditions that obtained in one run, altered this pattern. In general, however, the slower rate of wealth accumulation previously noted for these agents meant that they were unable to attain the increase required by the third or fourth year.

Type 8 also lost ground very rapidly in this version, the period between 3 and 4 years often proving critical for these agents. 10 years was the upper limit found. This was for Type 80 agents, who tended to outlast both Types 81 and 82. Levels close to this were rare, however. Again, those individuals surviving longest were those of the sub-type with K_α close to 1.

Type 7 agents were no longer able to maintain their percentage of the market. We recall that, apart from Example 3, these had the dominant share in Version 0. During the early stages of a run, Types 70 and 72 frequently increased their holdings. A common pattern was that this increase reached a peak, sometimes as early as 2 years, sometimes as late as 10, after which both types invariably lost ground, this loss coinciding with the increasing ascendancy of Type 50. The share of Type 71 tended to become negligible at an early date, but they persisted in the market longer than agents with both precision parameters greater than 1.

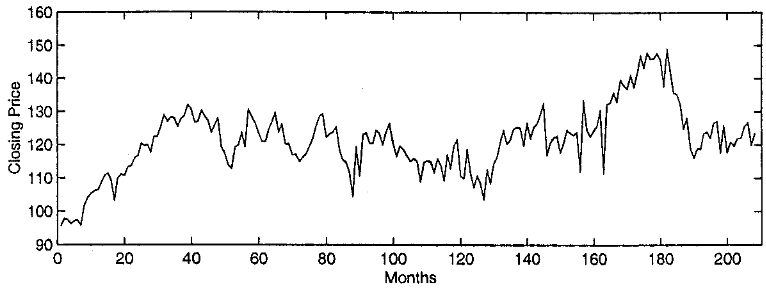

Figure 7.6: Monthly Price Series for Example 4.

While Types 50 and 40 increased their shares of the market, those with $K_\alpha < K_\beta < 1$ but higher values of K_μ did not perform nearly as well. Those with $K_\mu > 1.05$, Types 41 and 51, made little impression on the market, maintaining their share for a time but ceasing to take part altogether by 10 years in the average run. Agents with $0.95 \leq K_\mu \leq 1.05$ performed better and were notable for the achievements of some individuals of these types. Type 42 was usually able to maintain its initial holding with temporary increases occurring, at times substantial but not able to be sustained. Type 52, on the other hand, tended to double its initial share but no more. With these values of K_μ, however, improvement was slow, remaining constant over long periods. These types did not show the characteristic peaking and subsequent decline of those agents with the same values of K_μ but where K_β is above 1.

The details of a typical run are now examined.

Example 4

Parameters $(n1, \lambda_1, \lambda_2)$ in this run were set at (25, 0.3, 0.35). Distributions over the agents for the K-parameters for this example were taken from a lognormal distribution with a variance of 0.04 with the mean values of $(K_\mu, K_\alpha, K_\beta)$ set at (1.0332, 0.9245, 1.1386) at the beginning of the trial. Mean values after 16 years trading were $(0.83, 0.49, 0.84)$ showing a substantial change. The resulting distributions of $(K_\mu, K_\alpha, K_\beta)$ are shown in Figure 7.7. The accompanying price series is shown in Fig 7.6. Notice how different these distributions are from Example 1 of Version 0 shown in

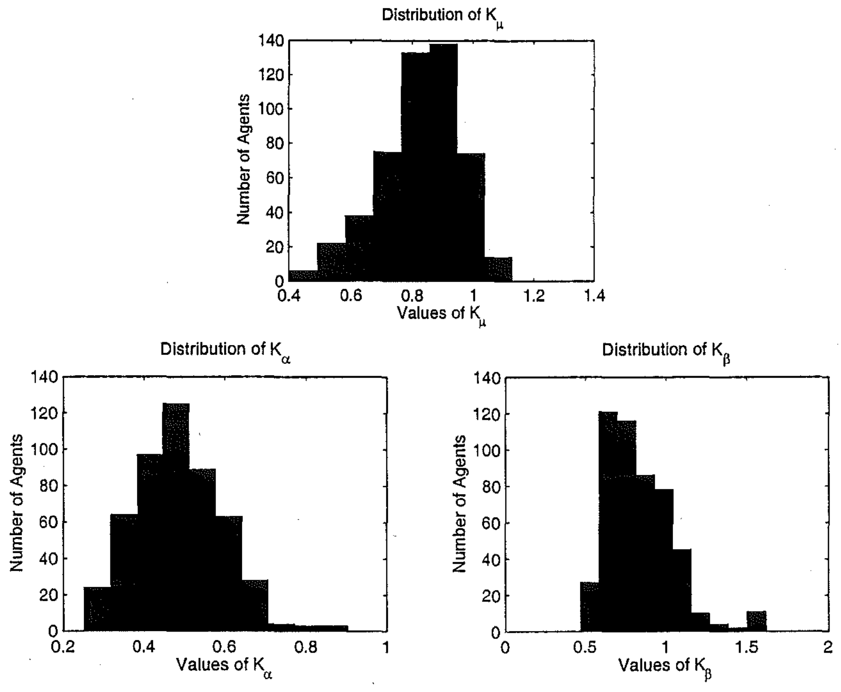

Figure 7.7: Distributions of K_μ, K_α and K_β for Example 4.

Figure 7.2.

In this run, the majority of Type 3 agents left the market after trading for a single year. A few individuals remained for 5 or 6 years. Numbers for this type were too small to make significant distinctions between sub-types, but it was noted that those with lower values of K_μ were inclined to continue trading for the longer time periods.

Type 81 survived until the 7th year, Type 82 until the 8th and Type 80 until the 10th year. The latter was then represented by a single individual who nonetheless attained the second highest level of wealth recorded at this time. Apparently, under these rules, this was no protection against an unfortunate transaction as that individual disappeared from the market the next year. Type 71 was represented until the 15th year, although the proportion had declined considerably between 5 and 10 years after which time it was very low.

By the conclusion of this run, Types 50 and 40 together comprised 64%

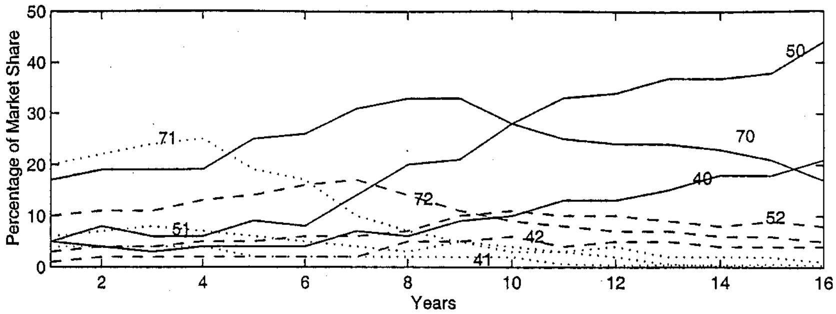

Figure 7.8: Changes in Market Shares for Example 4.

of the market population, the share of Type 50 alone being 44%. The only other significant players were Type 70 with 17% and Type 52 with an increased share of 8.2%. Remaining types still in the market were all below 5%, although it is interesting to note that the mean wealth of these small groups was comparatively high at this point of time. A chart showing the progressive market shares of the major players is given in Figure 7.8.

Characteristic of simulations using this version is the initial increase and subsequent decline in market share of Type 7 agents, seen in this run to be leading players for the first five years. It is recalled that there was no decline in Version 0. After this, Type 50, and later Type 40, exceeds these types. Type 70, falling more slowly, is passed at 10 years. This movement of Types 50 and 40 relative to their closest competitor, Type 70, is a pattern repeated frequently in other runs. Type 50 very often gained the highest share by 5 years and once before 2 years, in a run with $(n1, \lambda_1, \lambda_2) = (50, 0.01, 0.01)$.

It is interesting to note that much of what is being said here applies equally well to Example 3, which was run in the more tolerant atmosphere of Version 0, although presenting a pattern that occurred rarely in that context. The increases in Types 50 and 40 were of comparable magnitude, but no types were completely forced out of the market. The presence, in Example 3, of more agents with higher values of the precision parameters kept these distribution means considerably higher than for any run for Version 1. Percentages of market shares from the two runs are summarised in Table 7.2.

Type	Percentage of Market Share		
	Initial	Example 3	Example 4
3	7.2	0.8	-
4	10.0	22.2	24.6
5	14.8	49.8	53.4
7	46.0	25.8	22.0
8	22.0	1.4	-

Table 7.2: Comparison of Final Market Shares in Examples 3 and 4. These percentages were recorded at 16 years.

The presence, in Example 3, of more agents with higher values of the precision parameters kept these distribution means considerably higher than for any run for Version 1.

7.3.2 Comparative Profits

Version 1 resulted in a very wide spread of portfolio values in the experimental set. The number of successful agents at each 6-monthly test of profit levels was much lower than for Version 0, frequently with more than half of the set needing to be replaced. On the other hand, high levels of profit were common. The illustration in Figure 7.9 was taken from a typical trial with $(n1, \lambda 1, \lambda_2) = (25, 0.3, 0.35)$. The simulation was run for 16 years, at which time the price was \$97.29. Market shares for this run are shown in Table 7.3. Final mean values of $(K_\mu, K_\alpha, K_\beta)$ were (0.81, 0.63, 0.82).

Runs with this version frequently produced some very high portfolio values, so that there is a need to exclude such outliers from illustrative graphs for expositional practicalities. In this example, there were 11 individuals with fortunes in excess of \$50,000. It is appropriate to point out that, at 16 years, the required level is \$43,657, well below the portfolio values of these agents. Most agents, however, do not have to attain this level, since they have been in the market for a much shorter time. The histogram for the Experimental Set shows only those levels below \$50,000. These results reflect a considerable movement of capital and shares to agents of

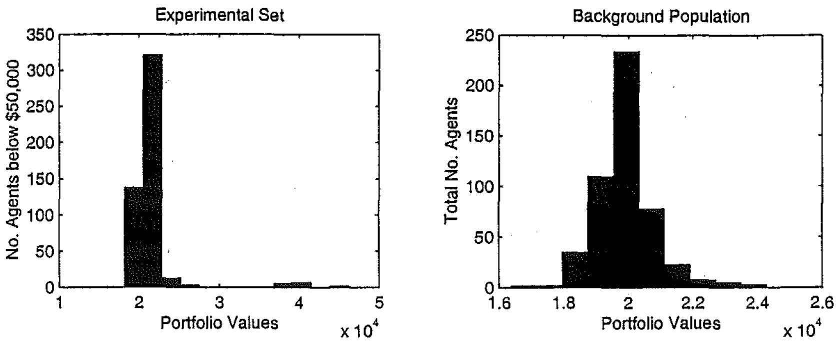

Figure 7.9: Distributions of Wealth (below \$50,000) for a Typical Run of Version 1. Portfolio Evaluation was made at 16 years at the price of \$97.29. Note that some very high yields above \$50,000 portfolio value are excluded from the histogram of the Experimental Set as explained in the main text.

Type	Percentage
40	36.2
42	1.2
50	57.8
52	3.2
70	0.8
72	0.8

Table 7.3: Percentage of Market Shares for the Run Illustrated in Figure 7.9.

the Experimental Set but the high turnover of agents in this set noted above. A large proportion, being comparatively new traders, have had little time to increase their profits and some of these may go on to reach high levels. At this point, however, there were 61 individuals of the Experimental Set with levels below \$20,000. The mean value for the background population was only \$19,891. 50% of the very high profits were for agents with values of $K_\mu > 1$ in the centre range, that is, with the label of '2', one of which was a Type 72 agent. None of these, however, could be said to be strictly in the "rational" class with values very close to 1. However, in some runs of this version, the high values were found to be held by agents who were among a very few survivors of their type.

7.3.3 Summary

The main features of this version can be summarised as follows:

1. A more pronounced move towards the left affecting all K-parameters.
2. A changing population in which Types 3 and 8 are unable to compete after the first few years; in which the largest market shares are held by Types 50 and 40, and in which Type 70 and those agents of Type 52 with K_μ-values below 1 give a variable performance.
3. Improved levels of total wealth overall; some very high values occurring; but with a very great disparity in wealth achievement among agents.

7.4 Version 2

7.4.1 Distributions and Market Shares

The removal of the opportunity for an agent to delay his decision to leave the market still further increased the rate of the developments noted in the last two sections. Agents with both precision parameters above 1 disappeared even earlier. The difference was particularly noticeable for Type 8, which was regularly found to hold less than 5% of the market share after the first year. Even this small amount was usually lost by the 4th or 5th year. The careers of Type 3 agents were similarly short-lived, the agents having largely disappeared after the first year.

Types with low K_μ were again dominant. Type 50 tended to increase rapidly and, at times, rose to a high percentage of the population. This increase occurred in all simulations. As with other versions, there was a wide range of results. Nine runs of the version were performed. In some runs, Type 7 agents increased or maintained their share for several years before disappearing. In one run, combined holdings of Type 7 still remained at 45% at 6 years, with Type 71 holding the largest share of these although beginning to decline at this point. In this version, these successes were short-lived, with no Type 7 group able to maintain its initial level over extended periods as was the case in Version 0, or even being capable of remaining in the market at all in the longer term. Type 40 proved to be variable. Frequently, a pattern similar to that of Type 50 was followed, being a steady rise accentuated by rapid advances coinciding with losses on the part of agents of Type 7. In a few runs, this rise led to a dominance of Type 40. In most trials, however, Type 40 did not gain the high share of Type 50. At times, in fact, progress for Type 40 was slow with market share barely retained over a long period.

The most commonly occurring outcome was one in which Type 50 held the greater share in a market dominated by Types 50 and 40. In these, Types 70 and 52 tended to retain some of their holdings for an extended period, occasionally as long as 12 years, but these were usually low compared to the initial shares of these types. This outcome is now illustrated by a further example.

Example 5

In this example, the program ran for 16 years resulting in a partition of market share between Types 50 and 40, with Type 50 holding 64%. The monthly price series for this run is graphed in Figure 7.10.

Type 3 agents survived only for the first year and Type 8 for 2 years. After 4 years, Type 41 had only one or two representatives who finally withdrew by 7 years. The few other agents with $K_\mu > 1.05$ were not doing well and were finally replaced between 11 and 12 years. Nonetheless, agents labelled subtype '2', ($0.95 \leq K_\mu \leq 1.05$), although decreasing in numbers, were showing some good individual performances in this run, averaging 50% of the highest twenty levels of wealth. The last agent of Type 72, however, left the market at 13 years and that of Type 52 a year later. Type 42 continued to be represented until the end of the run by a single agent with a K_μ of 0.98 and, in the final check, the third highest wealth level of the experimental set.

Since the second year, Types 50 and 40 had both been increasing their market shares rapidly, in particular, Type 50. However, although Type 40 had started more slowly, its market share doubled to a peak of almost 40%

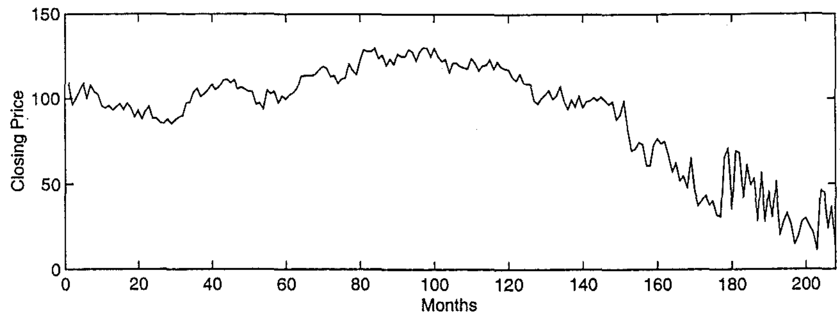

Figure 7.10: Monthly Price Series for Example 5.

of the total between 10 and 14 years. During this time, daily prices became increasingly volatile and finally fell to the low level of \$8. The parameter means of $(K_\mu, K_\alpha, K_\beta)$ were recorded at the end of the run at (0.68, 0.46, 0.7).

Although the dominance of Type 50 proved to be the most likely outcome, the occasions when it was equalled or superseded by Type 40 are interesting. Two runs, lasting for 14 and 16 years resulted in the high proportions for Type 40 of 92% and 98% respectively. The only other surviving group in these runs was Type 50. One of these runs is described in the next example.

Example 6

Progress charts for Examples 5 and 6 are given in Figure 7.11 and the price series for Example 6 in Figure 7.12.

In this 14-year run, the share of Type 50 exceeded that of Type 70 as early as the second year after which it retained the highest share until the ninth year when it suddenly fell rapidly to its 14-year level. At the same time, Type 40 continued to increase.

At the conclusion of the first year's trading, only two individuals of Type 3, both Type 30, remained in the market. These withdrew during the next 6 months. Type 8 fell from its original 22% to 0.4% after 2 years but was

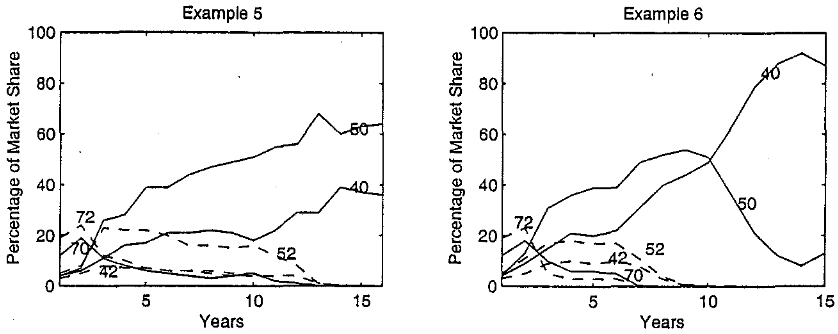

Figure 7.11: Comparison of the Patterns of Change in Market Shares for Two Examples of Version 2.

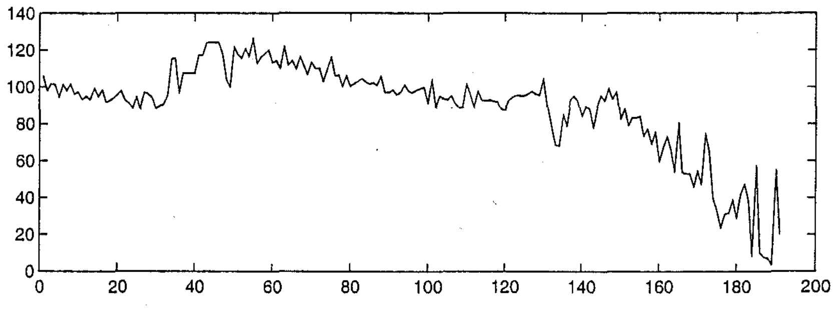

Figure 7.12: Monthly Price Series for Example 6.

represented for a further two years by a single individual with one of the highest levels of wealth.

Of the remaining agents, those with $K_\mu > 1.05$, whose share had fallen throughout, finally left the market between 5 and 6 years as did Type 70. The latter had followed their usual pattern by increasing their market shares briefly during the first year. Types 42 and 52 remained until the 8th and 11th years, respectively, both having maintained an increased share until the 5th year.

(Interestingly, in the 16-year run, the same pattern occurred with the reversal occurring also at 9 years, followed by a rapid collapse on the part of Type 50 and Types 51 and 70 losing their last remaining representatives at about the same time.)

The values of the precision parameters for this run fell rapidly, passing those of the average run of version 1 between 6 and 8 years. In this example, K_μ also fell very low. In the 14th year, means for the three parameters, of which the distributions are shown in Figure 7.13, were (0.44, 0.31, 0.46). As can be seen from the graphs, some very low individual values occurred for each of the parameters. The maximum value of K_μ at the conclusion of the run was that of a Type 40 agent who had accumulated a portfolio valued, before the crash, at $740,000.

This growth on the part of Type 40, however, does not appear to be a tenable situation. In a number of experimental runs, as in this example, it coincided with a period of excessive volatility with very few agents being successful and a final collapse in prices concluding the run.

Note: The further effects of price fluctuations of the scale that occurred in many of the runs for Version 2, and illustrated in Figures 7.10 and 7.12, and the possible recovery measures that could be put into operation were considered to be outside the scope of this work and no study of these has been attempted here. The patterns that have occurred are sufficient to indicate that these more extreme conditions of accountability appear not to be optimal in their present form neither for most individuals nor for the

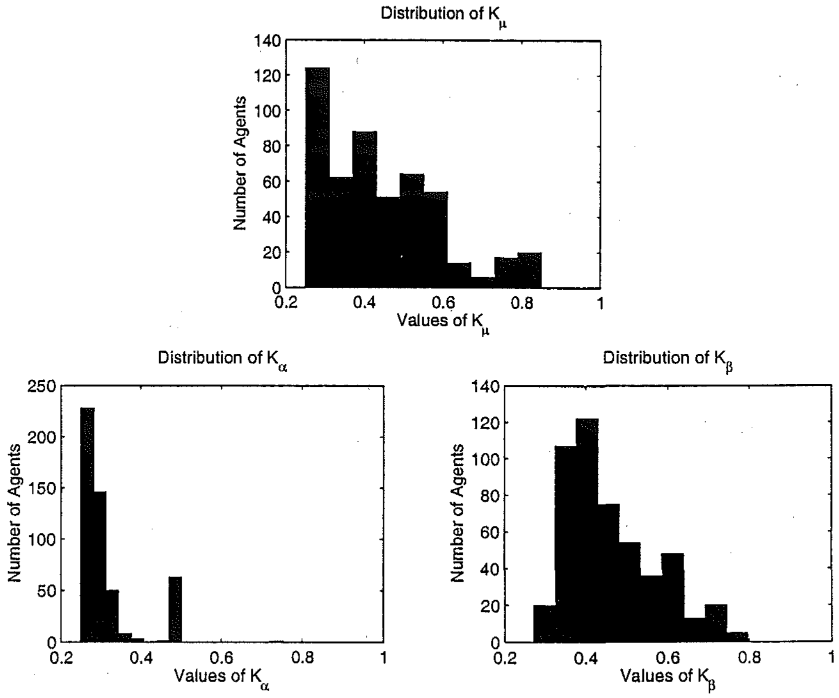

Figure 7.13: Distributions of $(K_\mu, K_\alpha, K_\beta)$ for Example 6.

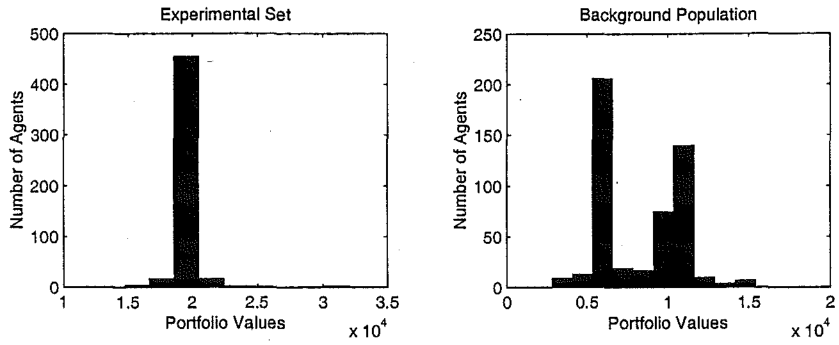

Figure 7.14: Distributions of Wealth for a Typical Run of Version 2. Portfolio Evaluation was made at the price of \$8.2 at 16 years.

operation of the market as a whole.

7.4.2 Profitability in Version 2

In spite of the fact that the rigid constraints imposed in this version result very frequently in crises in the market accompanied by rapidly declining prices, profits are still made by the experimental set. Results recorded at the time of the collapse of prices to \$8.2 described in Example 5 showed the evolving group to be more successful in maintaining the values of their portfolios than the rest of the market population. Comparative histograms for these values are shown in Figure 7.14. Although all values were necessarily low, the mean of the Experimental Set was \$18,871, with the maximum level at \$33,975 and the minimum at \$14,712. The background population, on the other hand, had a mean of \$8,236, with the highest recorded level of wealth being \$15,417 and the lowest, \$2,812.

7.4.3 Summary

This version showed several common features:

1. A strong movement towards the left in values of K_μ, K_α and K_β.

2. A population usually dominated by Types 50 and 40 in varying proportions.

3. The development of a long-run situation in which prices became more and more volatile, leading to a market collapse.

4. Generally improved wealth levels of market participants relative to the general population, dominated now by "losers". Even when the market crashed, the population of the Experimental Set maintained the value of their portfolios to a much greater extent than the traders of the background population.

7.5 A Comparison of Market Shares

In order to see the above results in perspective, Table 7.4 lists the percentage of the population belonging to each the groups studied for three of the examples discussed, one example being taken from each version. Percentages are shown both at 10 years and at 16 years, so that comparison can be made not only of the absolute levels between the different versions but of the nature of the changes over time occurring within these versions.

The above examples are chosen to show the range of possibilities found in the three versions. It is recalled that the most common result for Version 0 was that of Example 1, illustrated in Table 7.1. In more typical runs like this, Type 7 frequently maintained a level close to its initial share of the market. In these runs, it is also noticeable that the percentage of agents with both K_α and K_β above 1 was also much higher. In the example shown above, Type 7 agents increased their market share to 54.4%, the highest recorded for this type, and Types 3 and 8 fell to lower levels. The example for Version 1 is a typical one as is that for Version 2 where all runs showed similar extreme effects such as the dominance of Types 40 and 50 and a high degree of instability and price volatility.

	Version 0		Version 1		Version 2	
	Example 2		Example 4		Example 5	
Years	10	16	10	16	10	16
Type						
30	0.4	0.4	-	-	-	-
31	0.6	0.4	-	-	-	-
32	0.8	0.8	-	-	-	-
40	5.6	5.0	10.0	20.6	25.0	35.6
41	4.6	4.6	1.4	0.4	1.2	-
42	1.2	1.0	6.0	3.6	4.0	0.2
50	11.2	10.4	27.8	43.6	51.2	64.2
51	12.2	13.0	4.0	1.4	1.2	-
52	1.4	1.4	10.8	8.2	12.6	-
70	22.4	23.8	27.6	17.0	2.8	-
71	23.8	24.0	2.6	4.8	0.2	-
72	8.2	8.2	9.2	-	1.4	-
80	2.6	2.2	0.2	-	-	-
81	2.6	2.4	-	-	-	-
82	2.4	2.4	-	-	-	-

Table 7.4: Comparison of Final Market Shares for Examples 2,4 and 5.

Chapter 8

Conclusion

In Chapter 3, a range of Information Processing Rules was described. These rules arose from the varying traits that could be attributed to the individual traders in a speculative market. The study that followed has therefore been one in which the subjective opinions and decisions of these traders has been stressed. The case presented was that the history of prices is not so much shaped by external forces, although these do play a part, as by the internal composition of the market itself. This could, of course, be interpreted as a reflection of influences such as the adherence of agents to a current theory of trading or such as the market architecture itself. But the thesis put forward here is that even more important than these factors are the attributes of individual agents, quantified in the simulation by the learning process vector, $(K_\mu, K_\alpha, K_\beta)$. These parameters have been incorporated in such a way that they ultimately affect the reactions of the agents to all other influences whether internal or external to the market.

As evidence from the simulations accumulated, both from runs with an unchanging population and from the evolutionary models, these attributes were found to form themselves into five broad groups.

Firstly, there were those who, holding as they did strong views on the futility of attempting to predict prices, decided early against engaging in speculative trading at all.

Secondly was a group who were never able to feel certain about the opinions they expressed. Even when some of these had the skills to predict next period's price accurately, the value of this forecast was spoiled by a wide bid-ask spread or by uncertainty whether to place an order or not. These were the agents of Types 3 and 8. Experiments showed that some of this group could perform well where pressure to achieve was minimal.

The third group was the largest initially entering the market, Type 7. For these, the posterior mean of the precision parameter π increased with time, marking a belief in the precision of their forecasts but with a widening variance about this belief. Since these precisions increased without bound, these agents had accuracy with a degree of flexibility. This group were also more successful where the requirements on performance were less strict.

The most confident of the predictions proved to be those of the fourth group, those agents with low precision parameters, K_α and K_β, described as Types 4 and 5. These agents had the narrowest bid-ask spreads and the lowest doubt factors. Where this confidence for the precision was combined with a high value of K_μ leading to a reliance on the most recent observations, this group was successful only in an environment with few restrictions and not always then.

Fifthly, were those who were in the same precision category as the fourth group, but whose low K_μ-values led them to hold increasingly to their prior opinions. The experiments have shown these agents, Types 50 and 40, as a group at least, to be the most adaptable to short-term constraints. To a large extent, this adaptability did not occur on account of the adaptability of individual agents. This was apparent from the high replacement rate. Rather, the group dominance meant that it was able to revitalise itself by replacing its members by new ones displaying more advantageous non-adaptable characteristics until they themselves needed to be replaced. In the simulation, this group adaptation mechanism is used rather than one in which each agent might review his own individual attributes.

In any environment, the evolutionary aim of a population is to adapt and, in so doing, to mould that environment where possible to suit its own characteristics. This aim tends to be a short-term process which may not always be effective in the long run. The stresses imposed by the simulated environments encountered by agents in this model have embodied three levels of regulation. In the first version, Version 0, agents were free to determine for themselves the speed of portfolio increase. Falls below \$20,000 would presumably cause debts to be incurred, a situation which could only be tolerated for short intervals. The other two versions move into the realm of accountability, where agents are expected to show evidence of an accepted level of performance which, in Version 2, is set over a very short-term horizon.

The first level of regulative constraint was appreciated by all those agents with values of K_β above 1. Some of these, Types 8 and 3, were not confident of predictability but were able to support periods of inactivity during which this confidence could benefit from experience. When prices fell, for instance, these agents tended to retain their wealth in cash balances. Type 7 agents, on the other hand, trading more frequently and overshooting in their forecasts during a price fall when K_μ was high, were able to survive the occasional faulty decision. The main characteristic of this environment was the development, after a comparatively short period of adjustment, of a stable population in which Type 7 agents maintained a percentage of the market share close to their initial level, although with some changes among the sub-types. Those of Type 72 usually lost in numbers to the other two types but, as individuals, they frequently performed remarkably well. It has been seen that, in Version 0, agents with $K_\alpha < K_\beta < 1$ and low K_μ do tend to increase in numbers initially. However, the general stability of the population, with the majority of agents satisfied with a slower rate of wealth increase, limited the rapid proliferation of these agents that was seen in other versions.

This stability was removed with the imposition of accountability restraints. The overall effect of these was to force the population into making short-term decisions to remain active or to leave the market. The arbitrary 5% increase rule, for instance, meant that a portfolio must be valued at \$24,310 by the 5th year – with a 6-month lag in the intermediate version. Otherwise, the agent must leave the market. As regulating criteria intensified so did the instability of the population. Higher numbers of agents were replaced at each assessment of profits, even where the market was dominated by one or two types of agents. The market-generating mechanism being used to replace agents ensured that new entrants were imitating the approach of the most successful of the traders of the immediately preceding period, thus perpetuating the dominant types.

The results of repeated simulations suggested that the setting of short-term goals did improve the overall profit-making ability of a population but that this same population possessed attributes that, in some circumstances, could lead to a collapse of the market. The population generated in an atmosphere of increased accountability consisted of individuals who were confident of the precision of their expectations and would not be swayed from this. They placed undue emphasis on their prior opinions and displayed very little reaction to current news or to developing trends. Since there were other traders in the market who were fully aware of the trading potential of this information, the market spread widened and prices became volatile while declining, leading to panic and collapse.

In this study, short-term constraints that were artificially imposed have been seen to foster a population of agents vulnerable to their own weaknesses. An interesting extension of subjectivity would be the formation of an evolutionary process in which a group of agents not only learned to adjust their own attitudes but also learned to construct a set of optimal medium-term goals for profit-making. The agents in such a market would be subjectively directing their own evolution. On the other hand, individualised or not, the regulative constraints used here together with the weightings on information form just one of the many possible approaches for a simulated market. Other attributes which distinguish individuals and which are difficult to assess empirically could be examined. For example, attitudes of optimism or pessimism could be incorporated by a modification of the utility function used in the maximisation of portfolio values or attitudes ranging from aggressiveness to passivity could be reflected in an individualisation of the various aspects of interaction between traders. With regard to the overall framework, more general questions could be addressed. In the present thesis, all traders begin with portfolios of equal size in a single stock. A distribution of sizes over the population would enable the possible destabilising effects on the market of institutional investors to be examined. An extension to several stocks and the inclusion of dividends would open up the question of portfolio diversification. Further, the market architecture could be extended to describe the mixture of order matching and market making that is more characteristic of actual markets.

The modelling of a set of traders is aimed to highlight the relevant attributes of those traders and to study the effects of their consequent behaviour as they interact with other traders in their daily activities. But also relevant is the structure within which these agents make their decisions, the type of market, the occasional shocks and, perhaps most importantly, the effect on the individual traders of the market atmosphere which they themselves as a group have generated.

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